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Glenn Paul Roehrkasse
Iowa State University

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BEEF CATTLE PRODUCTION FUNCTIONS AND ECONOMIC OPTIMA IN BEEF CATTLE RATIONS

by

Glenn Paul Roehrkasse

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Dean of Graduate College

Iowa State University
Of Science and Technology
Ames, Iowa

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INTRODUCTION

Agriculture in the United States is in a process of making a conversion from a cash and row crop system to a grassland system* (53, p. 313). This conversion has been, in part, stimulated by 1) and increased interest in controlling soil erosion and 2) acreage control programs which allow farmers to sucstitute in production forage crops for grain crops. Such incentives which increase the supply of forages relative to grains create problems of how to utilize these forages in the most profitable manner.** Of the various alternatives available, livestock enterprises could be expanded and/or forages could be substituted for grain in the feeding ration. With beef fattening enterprises, the range of substitution of forage for grain will generally be greater between feeding

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*Grassland farming is defined by Stallings (53, p. 315) to be "... a system based on adequate and intelligent use of grasses and legumes, a system in which the grasslands are an integral part of the cropping scheme; a system in which some areas, unsuited for cultivation, are converted to permanent grasslands; a system in which other areas are placed in crop rotations with a sufficient proportion of grasslands to protect the soil and give profitable and sustained production of the cultivated crops."

**It has been stated by Heady _et al_. (23, p. 452), and shown elsewhere by Heady (18), that whether a shift of land from grain crops to forage crops will increase or reduce total output depends upon "... (1) the extent and rates at which crops substitute for each other in the crop rotation, (and) (2) the extent and rates at which these crops as feed substitute for each other in the livestock ration.

systems than within any one feeding system.* Of course, the amount of forage and grain that will be utilized and the rate as well as the range over which one feed substitutes for the other will depend upon the specific beef feeding system selected.

Within any one beef feeding system, farm managers must choose from the various combinations of forage and grain the least-cost combination of feeds (ration) which will produce a given livestock product ready for the market at a time when the price is such as to maximize profits. While there is a tremendous amount of information available on livestock feeding, there still exists considerable doubt and confusion, part of which is due to the numerous beef feeding systems, as to the extent forages will substitute for grain in the feeding ration. Therefore, there is a need for additional information as to the technical and economic forces affecting the returns from different forage-grain rations fed under different beef feeding systems.

Nature of the Problem

Agronomic studies have shown that where soil erosion is an acute problem grass and legume crops are especially effec-

*Bradford and Johnson (7, pp. 271-273) give a listing of 13 different systems of operating a beef fattening enterprise. These different systems were set up by farm management research workers from the North Central States.

tive as a control measure. Erosion control problems are more acute on rolling to hilly soils than on gently sloping to level soils. Since soil erosion is less under grass and legume crops than under row crops, high-forage rotations are recommended in rolling and hilly soils in order to obtain comparable erosion control. In many agricultural areas, the soil loss resulting from a high percentage of the land in row crops emphasizes the need of a cropping system including grasses and legumes. In order to reduce soil erosion in these areas an increased acreage of grasses and legumes may be needed in the rotations.

When an increase in the acreage of forage increases the supply of forage relative to grain (which depends on how the grain crops and forage crops substitute in the crop rotation), then this introduces the problem of how to utilize the forage in a profitable manner. In order for a farmer to make a decision as to the most profitable utilization of forages requires that he have a knowledge of how forages, and especially pasture forages, substitute for grain in the livestock ration. The feasability of substituting pasture forage for grain will depend upon the feed price ratio and the rate at which the feeds substitute for each other, and the rate of substitution will depend upon the class of livestock fed and the particular feeding system selected.

One opportunity for utilizing pasture forages is a beef fattening enterprise where pasture forages and corn are used as the fattening ration. However, relatively little is known of how pasture forages and corn substitute for each other in a beef fattening enterprise, yet without such information it is impossible to determine which combination of forage and corn should be fed in order to maximize profits. Profit maximization in a beef fattening enterprise depends not only upon feed costs, but also upon the time of marketing. A pasture forage-corn ration that "minimizes costs" may not necessarily be the ration that maximizes profits since profits are also affected by the time of marketing. The main objective of beef fattening is to improve the quality of the beef cattle but during the fattening period both the quality and the price per pound of the beef cattle change. Thus, the farm manager is concerned with selecting the least-cost pasture forage-corn ration that will place the beef cattle on the market finished to a grade at the time when the expected market price will be such as to maximize profits.

Scope and Objectives of the Study

The experiment upon which this study is based was designed to provide data that could be used to estimate the beef cattle production function and substitution rates between two kinds of feed -- corn and fresh-chopped pasture forage (soilage). The experimental data provides for estimates of the substitution rates between the two feeds for various combinations of the two feeds (rations) and, furthermore, provides for estimates of the most profitable combination of feeds under particular economic circumstances.

Also, included in this study is a review of the economic and production relationships that are relevant to a study of this nature. These concepts are then used to develop and construct a theoretical model that may be used to obtain an economic solution to some of the problems in livestock feeding.

The empirical results of the study are restricted by the limits of the experimental data. The feeder cattle used in the experiment were approximately 850 pound good-to-choice feeder steers. Therefore, the estimated relationships derived in the analysis will not necessarily apply to all other classes and grades of cattle. Furthermore, the feeding period is limited to the pasture growing season since the rations fed in the experiment were restricted to various combinations of the two feeds -- corn and fresh-chopped pasture forage (soilage).

The beef feeding experiment, mentioned above, was conducted at two separate locations. The only difference in the experimental design between the two locations was that stilbestrol was fed in the rations at one location and not at the other. This difference confounds the effects of stilbestrol with location. Thus, any difference that may exist between the stilbestrol and non-stilbestrol rations cannot be attributed simply to stilbestrol. The beef feeding experiment may be treated, however, as two separate experiments conducted at two different locations -- one where stilbestrol was fed in the rations and the other where stilbestrol was omitted from the rations. However, any comparisons that may be made between the stilbestrol and non-stilbestrol rations have to be made cognizant of the fact that they are not directly comparable.

The specific objectives of this study are: 1) to determine the rates at which pasture forage and corn substitute in the beef fattening process, 2) to determine the rate at which such feeds are transformed into beef gains for different pasture forage-corn rations, 3) to determine the time required to produce different levels of gain for different pasture forage-corn rations, 4) to determine the quality of beef cattle (i.e., the grade) produced from various pasture forage-corn rations, 5) to estimate, under different price conditions, the feed-gain-grade combination that will maximize profits for the pasture season, 6) to compare the various rations with and without stilbestrol, and 7) to consider new functional forms for evaluating feed-gain relationships.

BASIC CONCEPTS

In order to provide the basis for the formulation of the logic that will be adapted to specific livestock feeding problems as posed in the preceding chapter, a review of the relevant economic theory and production relationships seems necessary. In this chapter these fundamental concepts will be presented, not necessarily in full detail, but rather to show the relationships that can be adapted so as to provide a theoretical solution to the pertinent problems. These concepts so adapted will provide the basis for the hypotheses and the analytical framework within which this study is conducted.

Production Function

The relationships between resource inputs and product output can be characterized by a production function. Thus, a production function is a mathematical expression describing this functional relationship. A knowledge of the production function is of primary importance for not only does it show the factor-product transformation but it also serves as the basis for the derivation of the marginal products for individual factors, the iso-product contours (isoquants), the isoclines, and the substitution rates between factors.

The production function in its most general form may be written as

$$(1) \qquad G = g(x_1, x_2, \ldots, x_n)$$

where G is the product output and the x_i's ($i = 1, 2, \ldots, n$) are the variable resource inputs. However, in the case where only two resource inputs are considered variable the production function in its most general form may now be written as

$$(2) \qquad G = g(x_1, x_2 \mid x_3, \ldots, x_n).$$

When the production function is written in this form it indicates that x_1 and x_2 are variable resources and that $x_3, \ldots, x_n$ are fixed resources (i.e., the former are allowed to vary while the latter are fixed in quantity during the production period). While equation 1 denotes the case where all resource inputs are truly variable, most nutritions studies are carried on under the assumptions of equation 2 (i.e., some of the resources are considered fixed in quantity).

Production surface

It is possible to represent the production function under the assumptions of equation 2, which is a two variable function, as a production surface. This surface represents the output obtained from various amounts and combinations of the two variable resources.

For illustrative purposes let G denote the pounds of gain per steer, x_1 denote the pounds of grain fed per steer and x_2 denote the pounds of forage fed per steer. The production surface may now be located graphically by measuring the pounds of gain per steer, G, on the vertical axis and the two feeds, x_1 and x_2, on the respective horizontal axis. This concept is illustrated in Figure 1.

For each point on the gain surface there exists a unique set of coordinates (x_1, x_2) on the feed plane. Any two points on the feed plane represent either the same amount of gain on the gain surface (i.e., they lie on the same isoquant) or they represent different levels of gain on the gain surface (i.e., they lie on different isoquants). Furthermore, any two points on the feed plane represent either different rations or different levels of feeding of the same ration.

Iso-product or isoquant curves

A horizontal slice of the production surface projected down on the feed plane gives the locus of the various combinations of the two feeds that will produce the same gain. This locus or curve describes the various combinations of the two inputs that may be used in producing the same quantity of output and is called an iso-product or an isoquant curve. In a similar manner a family of isoquants may be developed with each isoquant representing a particular quantity of gain. This concept is shown in Figure 1, but may be reduced to a two-dimensional figure such as in Figure 2. On the production surface in Figure 1, contours depicting increasing levels of equal gain are represented by a, b, c and d, respectively.

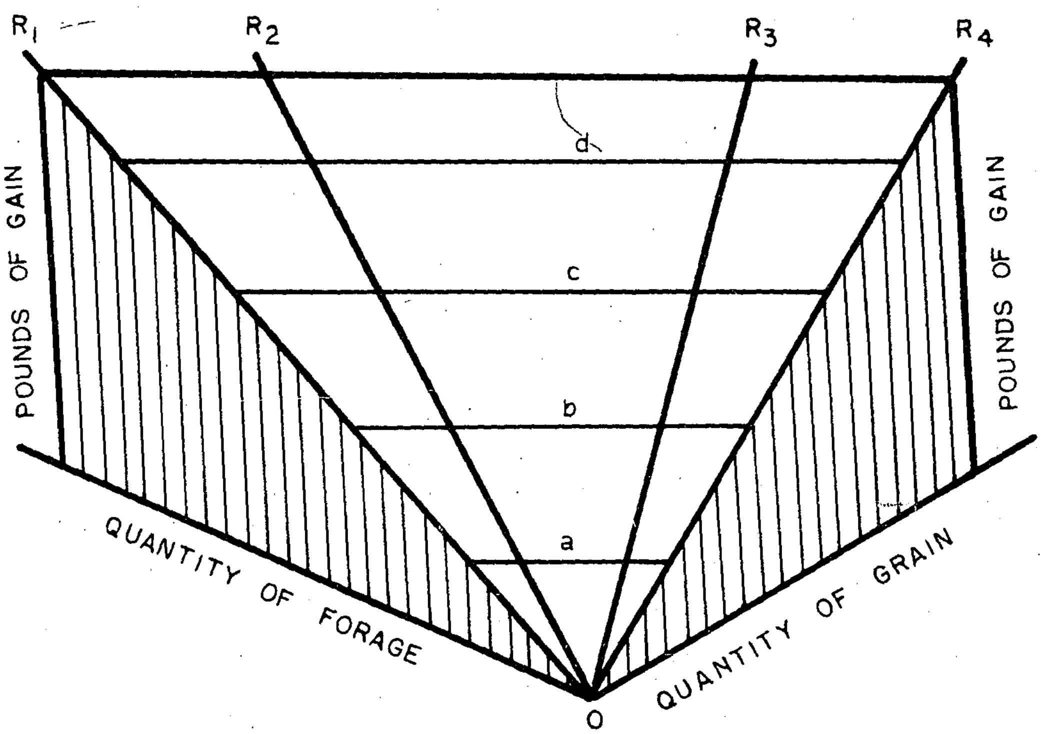

Figure 1. The production surface of a linear production function

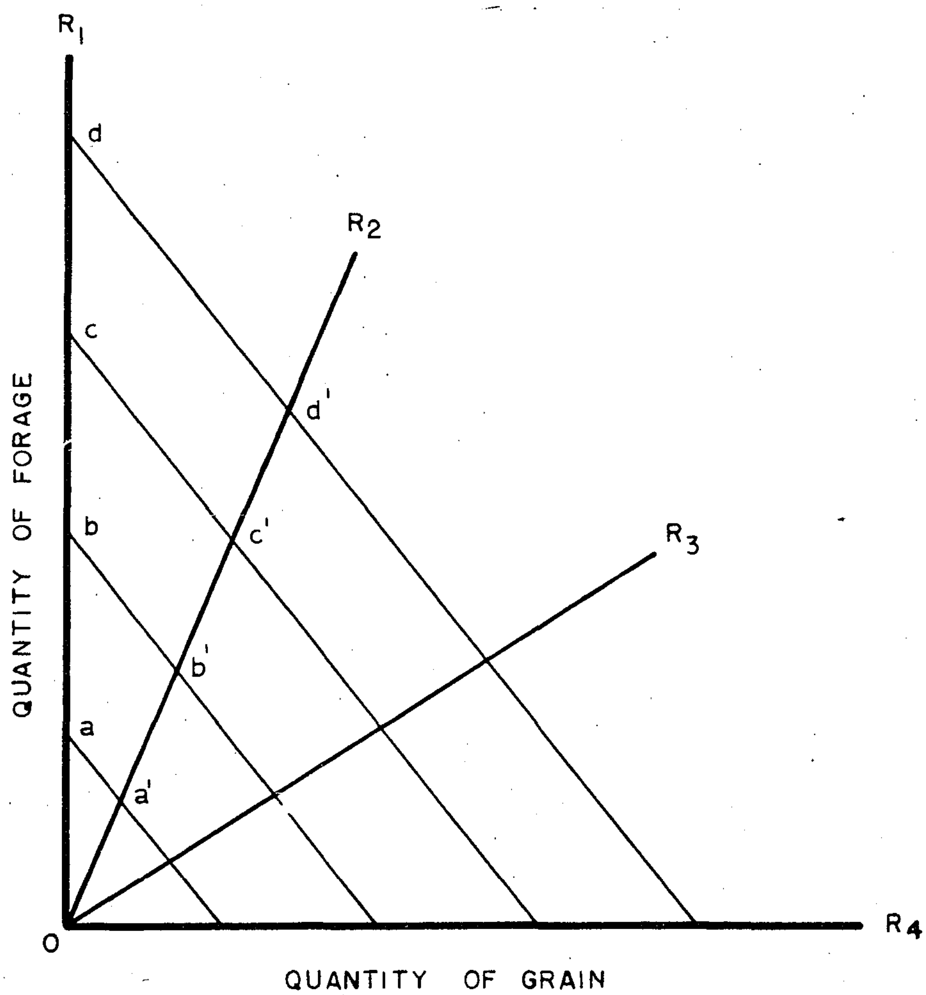

Figure 2. Linear isoquants for the linear production surface in Figure 1

The counterparts of these contours have been reproduced on the feed plane in Figure 2.

The particular relationship between the two feeds with gains constant depends upon the nature of the production function. That is to say, the particular shape of the isoquants depends upon the shape of the production suface. If, as in Figure 1, the production function is linear and homogeneous, the isoquants will be linear and parallel to each other.

The isoquant curves can be defined for a two variable production function, such as $G = g(x_1, x_2)$, by

$$g(x_1, x_2) = k \tag{3}$$

where k is a constant, representing the output value of a particular isoquant. Hence, all values of x_1 and x_2 that satisfy equation 3 trace out an isoquant in the x_1, x_2 plane.

Rate of substitution

The slope of the isoquant curve gives the rate at which one input substitutes for another input. Therefore, the marginal rate of substitution (MRS) may be defined as the amount one resource must be decreased, in order to keep the total output constant, as the input of another resource is increased by 1 unit. The marginal rate of substitution between two inputs, such as x_1 for x_2, may be thought of as being the negative inverse ratio of the marginal physical product (MPP) of the two inputs, i.e.,

$$(4) \quad MRS_{x_1} \text{ for } x_2 = -\frac{MPP_{x_1}}{MPP_{x_2}} = -\frac{dx_2}{dx_1}.\text{*}$$

In Figure 2 the marginal rates of substitution between the two feeds are constant since the isoquants have a constant slope. Even though two resources may substitute at constant rates they are not necessarily perfect substitutes, one for the other.

Ration lines

Total output may be increased, within the range of the substitutibility of the inputs, by either holding the quantity of certain inputs constant and increasing the quantity of all

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*By definition the MPP of x_1 and x_2 is, respectively,

$$(1) \quad MPP_{x_1} = \frac{\partial G}{\partial x_1} \quad \text{and} \quad (2) \quad MPP_{x_2} = \frac{\partial G}{\partial x_2}.$$

Since the total differential of the production function is

$$(3) \quad dG = \frac{\partial G}{\partial x_1} dx_1 + \frac{\partial G}{\partial x_2} dx_2$$

and since dG = 0 along any isoquant then

$$(4) \quad 0 = \frac{\partial G}{\partial x_1} dx_1 + \frac{\partial G}{\partial x_2} dx_2$$

and thus

$$(5) \quad \frac{\partial G}{\partial x_1} \Big/ \frac{\partial G}{\partial x_2} = -\frac{dx_2}{dx_1}.$$

Therefore, the MRS of x_1 for x_2 is defined by $-\frac{dx_2}{dx_1}$ (27, p. 47f).

inputs in some unprescribed manner. The special case where all inputs are increased in a fixed proportion is of special interest. In livestock feeding this special case is referred to as the feeding ration where the inputs are now different feeds and are increased in some predetermined fixed proportion. The ration lines are straight lines and must necessarily pass through the origin since the feed is fed in fixed proportions at all input levels. Furthermore, the ration lines will cross isoquants of increasing higher order as they extend outward as rays from the origin. This concept is illustrated in Figures 1 and 2 where OR_1, OR_2, OR_3 and OR_4 represent the fixed feed ration lines. Line OR_1 indicates a ration of feed x_2 while line OR_4 indicates a ration of feed x_1. Lines OR_2 and OR_3 represent constant ratios between the quantities of the two feeds x_1 and x_2.

Ration lines can be thought of as scale lines,* since they are the same lines. Therefore, it is possible with the aid of the ration lines to show if the feeds are transformed into gains at an increasing, constant or decreasing rate. For example, if the isoquants a, b, c and d, in Figure 2, represent 25, 50, 75 and 100 pounds of gain, respectively, and if the line segments $Oa' = a'b' = b'c' = c'd'$, then feed

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*Scale lines are used to show the returns to scale as all inputs are increased in the same proportion.

is transformed into gain at a constant rate.

The input-output or feed-gain transformation curve for each ration is another important relationship in livestock feeding which can be derived from the production function. Feed inputs along each ration line are combined into a ration in constant proportions. Therefore, if a vertical slice is made along a ration line, the total quantity of feed in the fixed ration is readily determined as is the total gain associated with the total feed inputs. This concept may be reduced from a cumbersome three-dimensional figure, such as Figure 1, to a two-dimensional figure, as illustrated in Figure 3. Since the production function in Figure 1 is linear, the feed-gain transformation lines in Figure 3 are also linear.

The transformation curves show the total quantity of gain forthcoming from various quantities of the given ration, while the slope of the curves indicates the rate at which the given ration is transformed into gain.

Isoclines

A line connecting all points of equal rates of substitution on successive isoquants is called an isocline. The pattern of a family of isoclines for a production surface depends upon the nature of the production function. Therefore, a family of isoclines, depending upon the nature of the

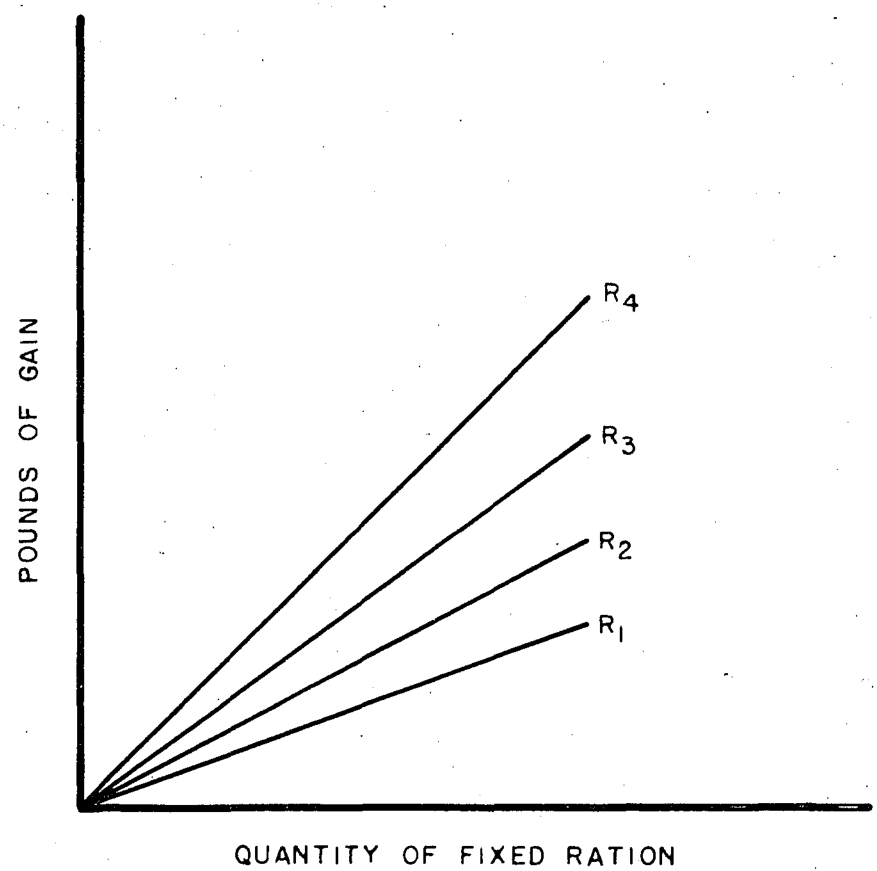

Figure 3. Feed-gain transformation curves from a linear production surface

production function, may be 1) linear or curvilinear or both, 2) they may extend outward from the origin or from the axes or both, 3) they may converge at some point in the input plane, indicating a maximum output, or 4) they may extend outward and never reach a maximum output. This concept is illustrated in Figure 4.

Certain isoclines within a family of isoclines are of special interest and, hence, have been given special names. The two isoclines that indicate the points on successive isoquants where the substitution rates between the inputs become zero, or where the inputs become complementary, are called "ridgelines", as illustrated in Figure 4. Thus, they define the area in which the inputs are substitutable and, furthermore, they define the area within which the decision is to be made as to the optimum combination of inputs.

The isocline that indicates the least cost combination of inputs for a given price ratio of inputs is called an "expansion path". The expansion path defines the combinations of inputs, given the price ratio of the inputs, that will give the least cost for successive levels of output. This optimum combination of inputs may change or remain constant for various levels of output depending upon the nature of the production function. When the optimum combination of inputs remains constant for all levels of output, the expansion path, the scale line and the ration line will coincide.

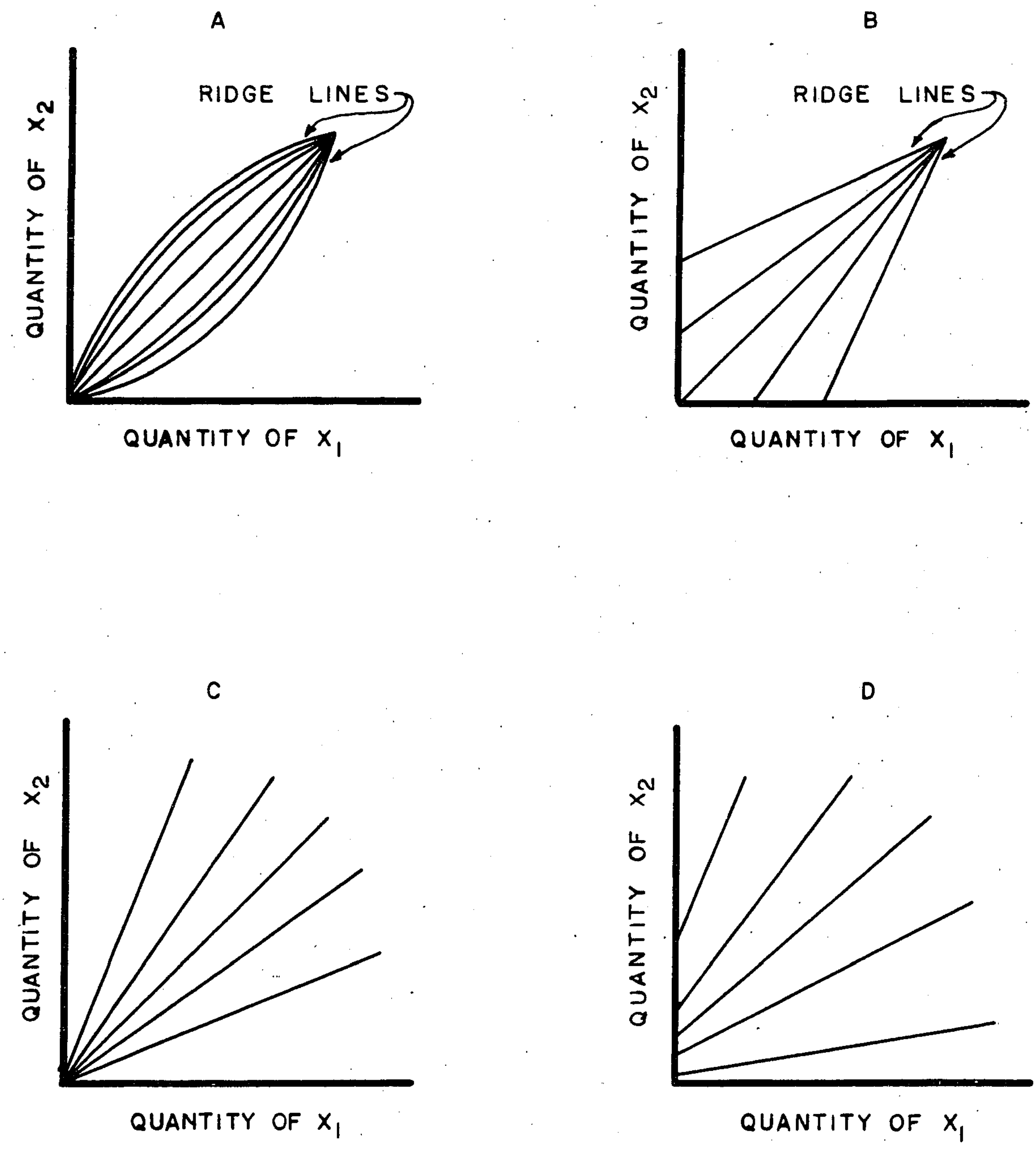

Figure 4. Alternative hypothesis regarding the shape of isoclines

Returns to scale

Returns to scale refers to how output responds when all inputs are increased in the same proportion. If output increases in the same proportion as the inputs, returns to scale are constant. Returns to scale are increasing if output increases by a greater proportion than the inputs and the returns to scale are decreasing if output increases by a smaller proportion.

Returns to scale can be easily shown for homogeneous production functions. A production function, e.g., $G = g(x_1, x_2)$, is homogeneous of degree s if

$$g(kx_1, kx_2) = k^s g(x_1, x_2) \tag{5}$$

where k is any positive number and s is a constant. Therefore, if inputs x_1 and x_2 are both increased by amount k output will be increased by k^s. If $s = 1$ returns to scale will be constant, if $s > 1$ they will be increasing and if $s < 1$ they will be decreasing.

The classical production function exemplifies all three stages of returns to scale. At low levels of input there exists first increasing returns to scale, then over some portion of the curve constant returns and finally at greater input levels decreasing returns.

The returns to scale concept differs from the law of variable proportions where only one input is allowed to vary while all other inputs are held fixed.

Least Cost Combination

Once the technical conditions of production have been determined, factor prices need to be introduced in order to determine the optimum combination of inputs for various levels of output such that the cost of production will be a minimum for such levels of output. That is, the ratio of factor prices is the choice indicator that is used to determine the optimum combination of inputs for various levels of output.

Where the market structure is relatively competitive as in agriculture, a single intrepreneur cannot significantly affect the market price of factors by the quantity he buys nor can he affect the market price of products by the quantity he sells. Therefore, the market prices of factors used in production will remain unchanged and independent of the level of output a farmer may choose to produce. Similarly, the market prices of the products he has to sell will remain unchanged and independent of the quantity he places on the market.

An isocost line is defined by the various combinations of inputs that may be purchased for a given total cost. The slope of this isocost line is equal to the negative reciprocal of the input price ratio. A family of isocost lines may be generated by varying the total expenditure for inputs. The isocost lines will intersect many isoquants and will become tangent to one isoquant which will indicate the maximum amount of output that can be produced for a given total cost, and at the point where the two curves are tangent they will have equal slope. Since the slope of an isoquant is the marginal rate of substitution of one input for another in production and the slope of an isocost line is the inverse ratio of the input prices, then the point where the two curves are tangent defines the least cost combination of inputs for a given output. The locus of the expansion path may now be traced out by connecting the points of least cost on successive isoquants. This concept is illustrated in Figure 5 where C_1, C_2, and C_3 are three isocost lines of a family of isocost lines where $C_3 > C_2 > C_1$ and Q_1, Q_2, and Q_3 are three isoquants of a family of isoquants where $Q_3 > Q_2 > Q_1$. Points a, b, and c represent points of least cost for different levels of output and OE represents the expansion path.

If the prices of the inputs x_1 and x_2 are P_{x_1} and P_{x_2}, respectively, then the point of the least cost combination, for a given output, will occur when

$$(6) \quad -\frac{P_{x_1}}{P_{x_2}} = -\frac{\partial x_2}{\partial x_1} = -\frac{MPP_{x_1}}{MPP_{x_2}} = MRS_{x_1} \text{ for } x_2 .$$

If $\dfrac{P_{x_1}}{P_{x_2}} > \dfrac{\partial x_2}{\partial x_1}^{*}$, then $\dfrac{MPP_{x_2}}{P_{x_2}} > \dfrac{MPP_{x_1}}{P_{x_1}}$ so that it is

*Throughout the remainder of this thesis, the negative sign on the marginal rate of substitution will be omitted. This procedure is in accord with the general practice since the substitution rate is negative in all rational areas of production.

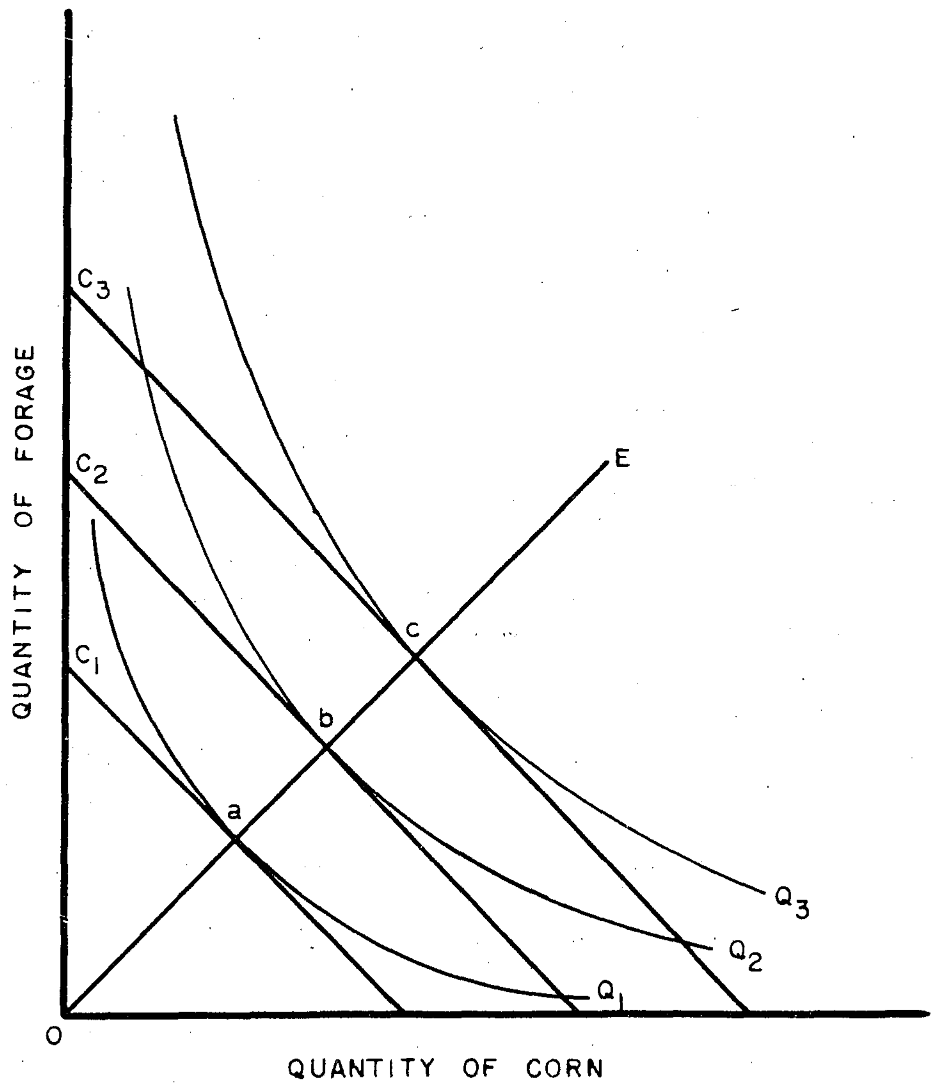

Figure 5. Illustration showing the relationship between the expansion path and isoquants and isocost lines

profitable, in the production of a given output, to substitute x_2 for x_1 until the ratio of the marginal physical products of a factor to the price of the factor are equal for all factors which is the least cost combination of inputs.

If $\frac{P_{x_1}}{P_{x_2}} < \frac{\partial x_2}{\partial x_1}$, then $\frac{MPP_{x_2}}{P_{x_2}} < \frac{MPP_{x_1}}{P_{x_1}}$, and so in this case it now becomes profitable to substitute x_1 for x_2 in the production process. However, if, as in (6),

$$\frac{P_{x_1}}{P_{x_2}} = \frac{\partial x_2}{\partial x_1}, \quad \text{then} \quad \frac{MPP_{x_2}}{P_{x_2}} = \frac{MPP_{x_1}}{P_{x_1}}$$

and there is no incentive to substitute one input for the other.

Since the least cost combination of inputs occurs on different isoquants where

$$\frac{MPP_{x_1}}{MPP_{x_2}} = \frac{P_{x_1}}{P_{x_2}} \tag{7}$$

the equation of the expansion path may be derived algebraically from (7) to give

$$x_1 = x(x_2, P_{x_1}, P_{x_2}) \; . \tag{8}$$

Profit Maximization

Any rational entrepreneur who is attempting to maximize profits, and is producing a homogeneous product at all levels of output, will carry on production with an input combination that lies on the expansion path. The mere selection of a combination of inputs that lie on an expansion path does not, of course, assure that maximum profit will be attained. However, for any given output the lowest cost of production will be that combination of inputs that lie on the expansion path corresponding to that level of output. Since profit is defined as total revenue minus total cost, then maximum profit will occur at that output where total revenue minus total cost attains a maximum. The output that maximizes profits can be shown to be the same output where marginal revenue is equal to marginal cost.* The cost of producing various levels of output, using the combination of inputs indicated by the expansion path, will provide a series of costs that can be used to plot a total cost curve. In a like manner, the revenue obtained from the output at the various levels of production will provide a series of revenues that can be used to plot a total revenue curve.

Total costs will be increased by the amount of the fixed costs which are the costs for the factors of production which were held fixed during the production process as explained in an earlier section. The total revenue curve may be assumed to be linear since the price of the output is not considered

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*The second order condition for profit maximization requires that the marginal cost curve is rising.

a function of the level of output.

The concept of profit maximization is illustrated in Figure 6. The level of output where profits will be maximized occurs at the output OQ' for at this level of output total revenue minus total cost is a maximum.

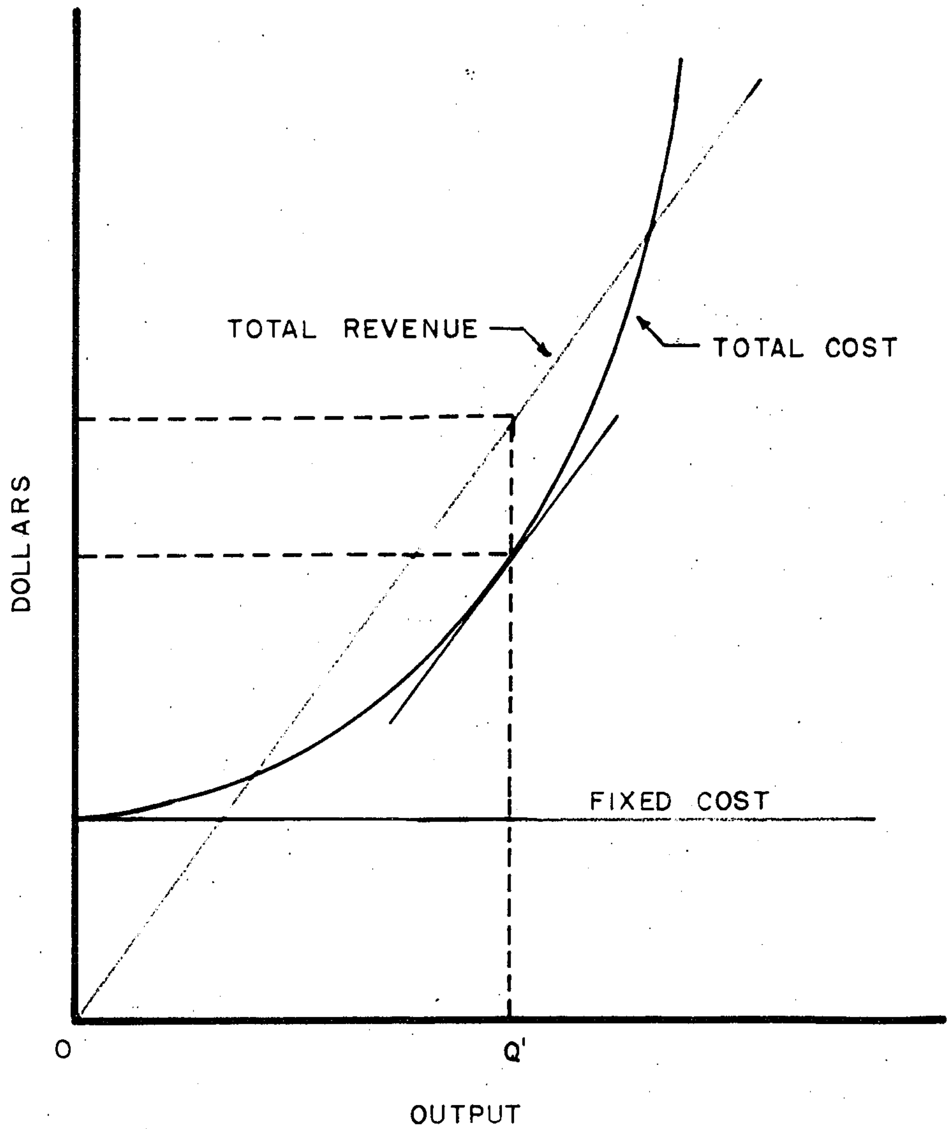

Figure 6. Illustration of the relationship between total cost, total revenue and fixed cost in the maximization of profit

A THEORETICAL MODEL

In the last chapter some of the basic economic and production relationships were examined as a basis for developing the logic necessary to construct a theoretical model to obtain an economic solution to some of the problems in livestock feeding. In the present chapter a theoretical model is developed to hypothesize logical production functions, isoquants, isoclines, marginal rates of substitution and related economic quantities that are needed for an economic solution of problems in beef cattle feeding.

Production Functions

The various classes of feeder cattle will not have exactly the same production function. According to nutrition theory, feeder calves require less feed per 100 pounds of gain than yearling feeders which in turn require less feed than two-year old feeders. Also, the nutritive requirements for fattening calves are different from those for yearlings or for two-year olds (46, p. 204; 48, p. 4). While the various classes of feeder cattle may have different production functions, nevertheless, the concepts presented in this section may well apply to all classes of feeder cattle.

The production function for feeder cattle will generally involve a large number of resource inputs, such as the production function

$$(9) \qquad G = g(x_1, x_2, \ldots, x_n)$$

where G is pounds of gain and $x_1, \ldots, x_n$ are resource inputs. However, for the purpose of this discussion only two feed resources, corn and forage, are considered as variable resources which can be varied in quantity during the production period. Even though the production is limited to two variable resources, the concepts that are presented can be applied to feeding problems involving more than two variable inputs. The production function with two variable resources may then be written as

$$(10) \qquad G = g(C, F \mid x_3, \ldots, x_n)$$

where C is the corn input and F is the forage input and the other variables, $x_3, \ldots, x_n$, are resource inputs that are fixed in quantity during the production period.

One possible production function for feeder cattle is the linear production function which gives a linear production surface as shown in Figure 1. Since the production surface is linear, it indicates constant returns to feed (*i.e.*, each successive increment of feed for a given ration produces similar increments of gain as illustrated in Figure 3). Under the assumptions of this model no feed is limitational so that gains can be obtained with a ration of only corn or only forage or a ration made up of any combination of the two feeds. The isoquants corresponding to this function, shown in Figure 2, indicate that the marginal rate of substitution between feeds is constant for all levels of gain (i.e., the marginal rate of substitution between feeds between rations is constant for any given level of gain). Furthermore, the marginal rate of substitution between feeds will be constant for any given ration regardless of the level of gain.

The least-cost ration, under the assumptions of a linear production surface, will generally be either a ration of only corn or only forage. The only time a least-cost ration could include both forage and corn, such as R_2 or R_3 in Figure 2, would be the special case when the isocost line had a slope identical to the slope of the isoquants and even under these conditions such a ration would not be preferred to a ration of either all corn or all forage under the least-cost criteria.

If the production surface is linear then there is no limit to the amount a feeder may gain. Therefore, if it is ever profitable to feed any ration it will certainly be profitable to feed an infinite amount of that ration.

While the production surface for feeder cattle may be linear over a portion of the surface, it appears rather illogical to assume the surface to be linear over all extremes. For to assume the isoquants are strictly linear with a constant marginal rate of substitution between feeds, assumes that the two feeds perform the same biological function and differ only in their effectiveness of producing gains unless the marginal rates of substitution between feeds is equal to one (63, p. 30). Furthermore, to assume that the feed-gain transformation curves are linear, as in Figure 3, assumes that for any given ration a feeder animal will put on gains as efficiently at the end of the feeding period as at the beginning of the feeding period.

The hypothesis that the feed-gain transformation curves are linear appears to be illogical on the basis of economic logic and experimental evidence. According to animal nutrition theory (46, pp. 180, 202) an animal requires more feed for maintenance per 1000 pounds of liveweight as it becomes fatter, partly due to the larger body surface and partly due to the fat condition itself. In addition an animal may actually consume less feed per 100 pounds of liveweight as it becomes fairly fat, therefore there will be less nutrients left for additional gains after having met the maintenance requirements. Hence, one would expect decreasing returns to feed as an animal becomes fairly fat.

Another possible production surface for feeder cattle is shown in Figure 7. The feed-gain transformation curves corresponding to this production surface are shown in Figure 8. The feed-gain transformation curves indicate that for any given ration there exists diminishing returns to feed. That is, each additional increment of feed, of a given ration, produces smaller and smaller increments of gain. The iso-

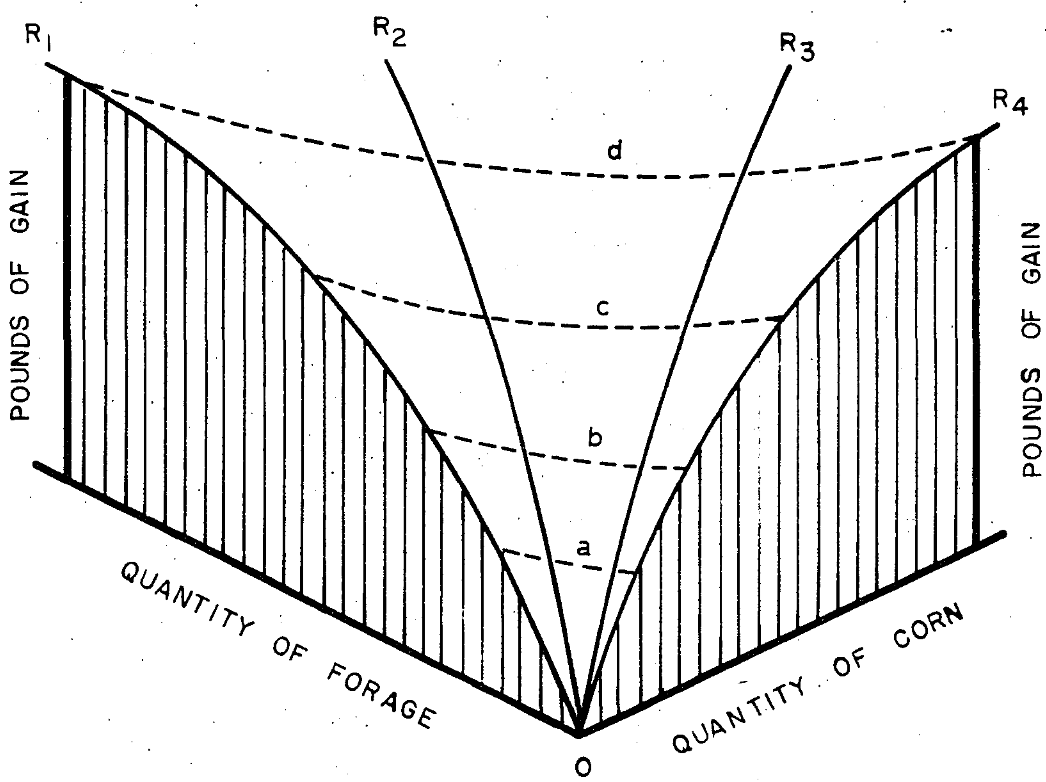

Figure 7. Production surface showing diminishing returns to feed

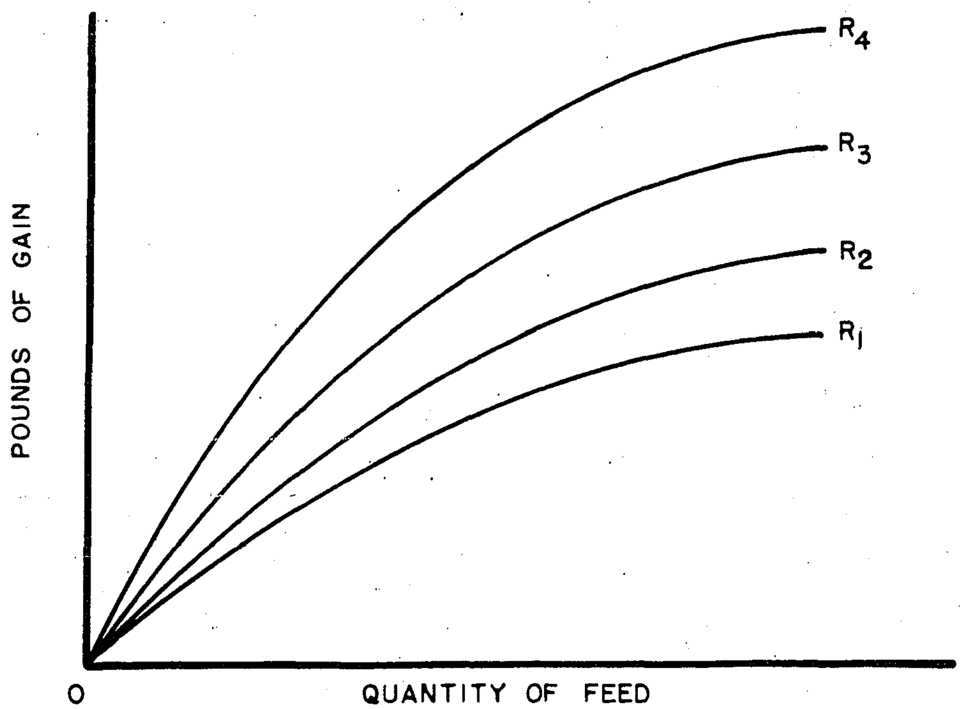

Figure 8. Feed-gain transformation curves showing diminishing returns to feed

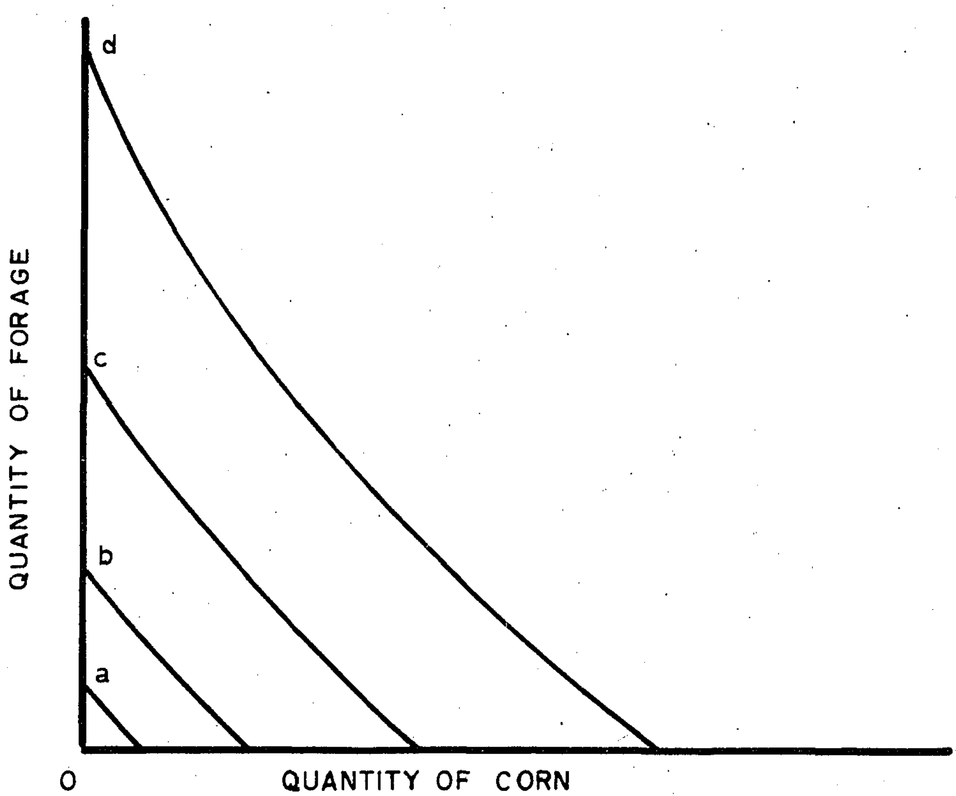

Figure 9. Isoquants for the production surface in Figure 7

quants for this function are shown in Figure 9. Since the isoquants are now convex to the origin and are no longer linear, as with the linear production surface, indicates that for any given level of output the two feeds substitute at diminishing rates. Therefore, the substitution rate between feeds will be different between rations for any given level of gain. Furthermore, since the isoquants intersect both axes assumes that neither feed is limitational (*i.e.*, gains can be produced with a ration of either only forage or corn as well as any combination of the two feeds).

Even though this latter production surface allows for diminishing returns to feed and is a more logical model than the linear production surface it is still quite unlikely that it adequately represents the feed-gain relationships of beef cattle feeding. While the exact nature of the feed-gain production surface has not been established a more logical model is shown in Figure 10.

Under the assumptions of this model there is a minimum amount of forage that must be included in the ration if gains are to be maintained over a long period of time. This assumption is consistent with nutrition theory (48, p. 6), for the need of at least a minimum amount of roughage in the fattening ration of beef cattle is well recognized. The requirement that at least a minimum amount of roughage must be included in the ration of beef cattle specifies the limiting

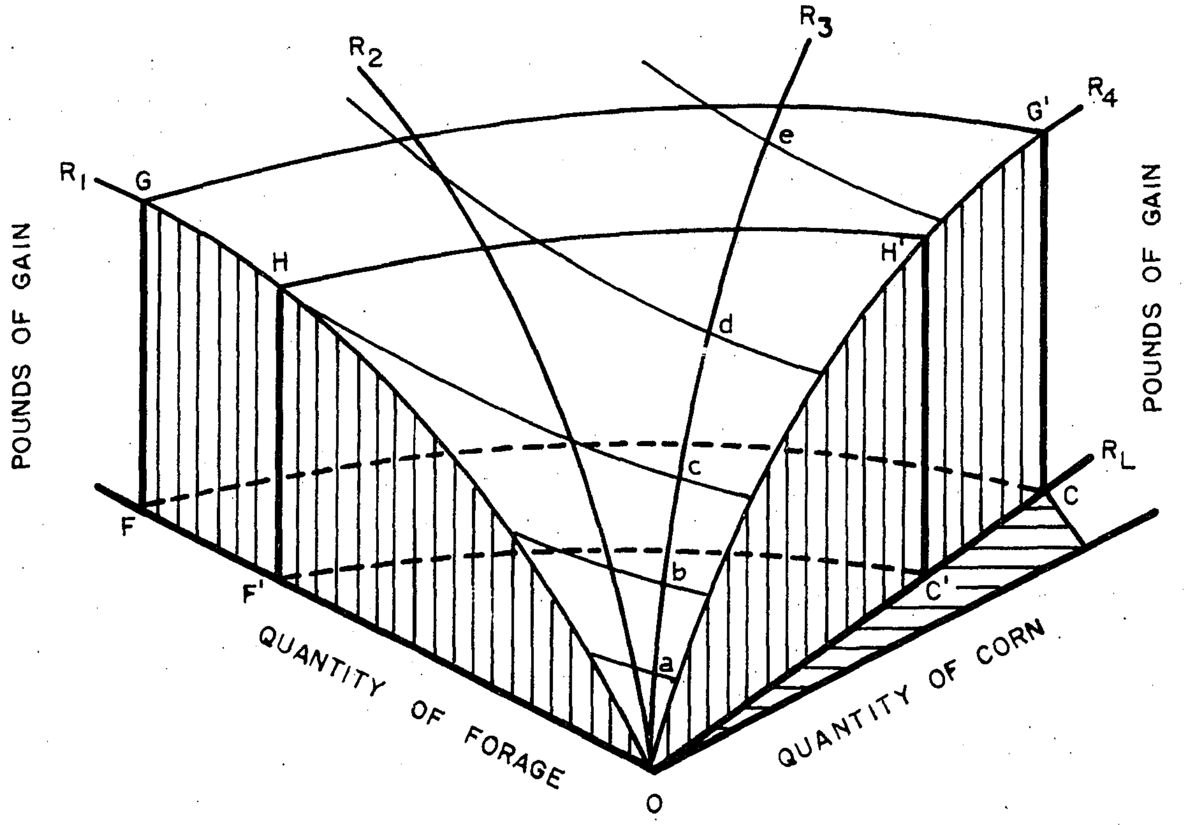

Figure 10. Production surface for beef cattle illustrating the physiological limiting ration and the stomach capacity

corn-forage ration that can be fed if gains are to be maintained. This concept is illustrated in Figure 10, where the physiological limiting ration is denoted by the line OR_L.

This model provides for diminishing returns to feed for all rations, as shown by the feed-gain transformation curves in Figure 11. Also, since the isoquants, illustrated in Figure 12, are non-linear and convex to the origin, the substitution rate between feeds will decline for any given level of gain. That is, the rate of substitution between feeds will be different for different feed rations for any given level of gain. Moreover, along any one ration line the rate at which feeds substitute one for another will also vary. This change in the rate of substitution between feeds along any one ration line and between rations for any one level of gain is supported by animal nutrition theory (46, pp. 203, 205). For the fattening of meat animals, most of the fat is generally formed from carbohydrates and the value of different feeds for the formation of fat depends upon the amount of net energy or total digestible nutrients in the feed. Furthermore, as an animal becomes fatter a larger proportion of the total nutrients in the ration is required for maintenance and consequently less nutrients remain for the formation of fat. Thus, as an animal takes on heavier weights the substitution rate between corn and forage would tend to change. Therefore, the expansion path or the least-cost

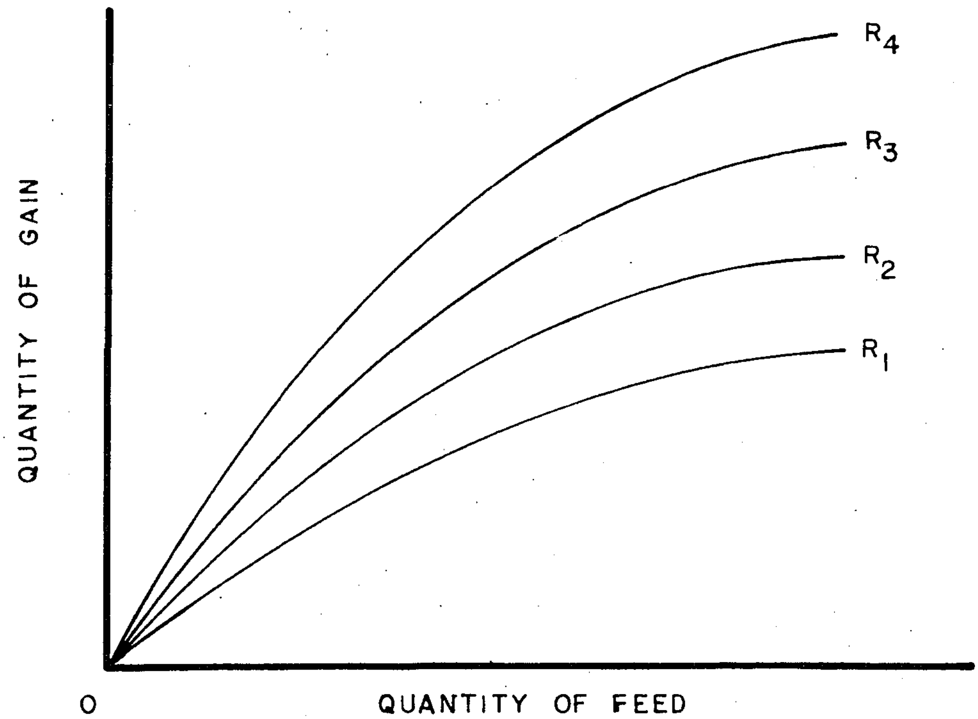

Figure 11. Feed-gain transformation curves for Figure 10

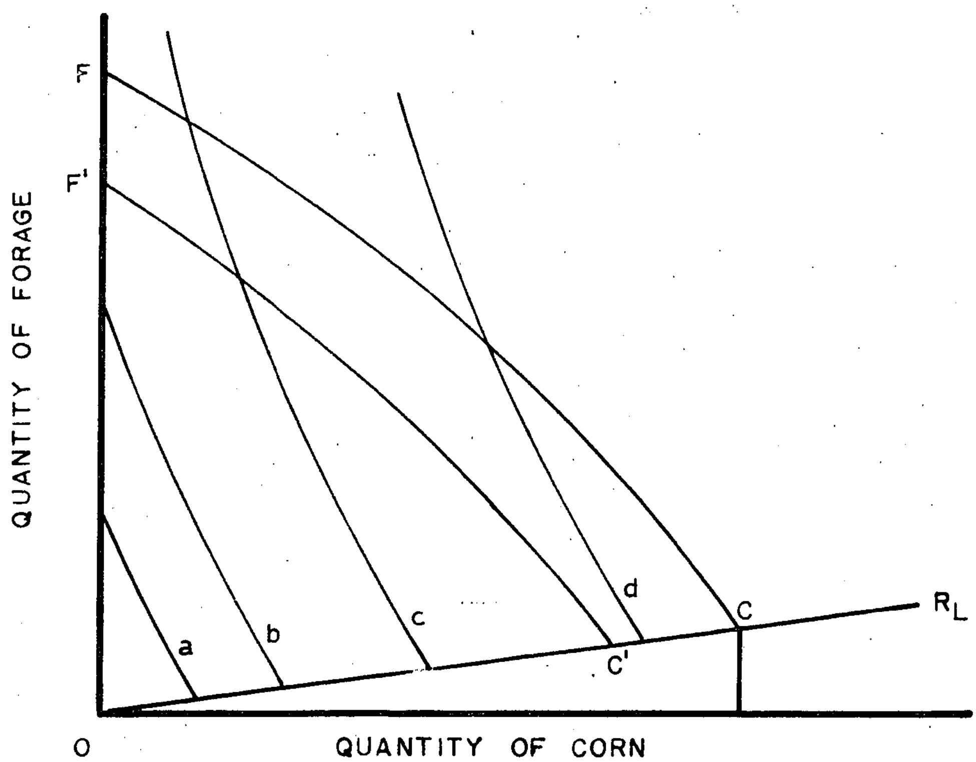

Figure 12. Isoquants and isotime curves for Figure 10

combination of feeds need not necessarily coincide with the ration lines. This would indicate that it is more profitable to feed a different ration for each level of gain.

The isoquants a, b, ..., e, in Figure 12, indicate the various combinations of feed that will produce a given level of gain, and the slope of the isoquants indicate the marginal rate of substitution of corn for forage. The slope of the isoquants must decrease as the feed rations approach the limiting ration OR_L if diminishing rates of substitution are to hold true.

The purpose of this section has been to present some alternative hypotheses as to the nature of the beef cattle production surface. The alternatives that have been presented by no means preclude the possibility that other alternative hypotheses exist (38). Nevertheless, the nature of the production surface in Figure 10 appears to be a logical hypothesis regarding the general nature of the beef cattle production surface.

The nature of the production function, which can be represented as a surface, needs to be established in order that production relationships such as the factor-product transformation, the substitution rates between factors, the isoproduct contours, etc., may be determined.

Least-Cost Rations

Once the production function has been derived it is possible to determine the ration or combination of feeds that gives the lowest cost for a specified level of gain. The least-cost ration, for a given level of gain, will be specified when the following condition is satisfied,

$$\text{(11)} \qquad \frac{\partial F}{\partial C} = \frac{P_C}{P_F}$$

where $\frac{\partial F}{\partial C}$ is the marginal rate of substitution of corn for forage and P_C is the price of corn and P_F is the price of forage.

Generally, the least-cost ration will fall between the physiological limiting ration and the all forage ration. That is, the least-cost ration will generally fall between these two limits as long as the gain isoquants are non-linear and the feed price ratio is less than the marginal rate of substitution between feeds for the all forage ration and is greater than the marginal rate of substitution between feeds for the physiological limiting ration.

Since the substitution rate between feeds will change along any given ration line, the least-cost ration for a specified level of gain will not be the least-cost ration for any other level of gain. That is, the point of intersection of the expansion path, which is an isocline, and an isoquant denotes the least-cost ration for that level of gain if only a single ration is to be fed for the entire feeding period.

A beef gain isoquant is by definition the locus of all combinations of the two feeds that will produce a given level of gain. However, any one point on the gain isoquant not only indicates the combination of the two feeds that will produce the given level of gain but also specifies the ration that will produce this level of gain. The time required to produce the given level of gain will be different for each combination of feeds along the isoquant. Consequently, the beef grades will also differ along any one isoquant. Thus, the point of intersection of the expansion path and a given gain isoquant not only specifies a certain level of gain and the ration that will produce this gain but also a certain grade of beef. By feeding the ration that passes through the intersection point of the expansion path and a specified level of gain results in the level of gain and the associated beef grade being produced at least cost. This ration will not, however, be the least-cost ration for any other specified level of gain. Therefore, if a single fixed ration is fed in order to produce some predetermined level of gain, then this ration will not be the least-cost ration if less or greater gains are to be produced.

If a feeding experiment is conducted by feeding several different fixed rations but for any one lot of steers they are all fed the same fixed ration for the entire feeding period, then it is not possible to determine what the effects will be on gains from changing the feeding ration during the feeding period. Any attempt to do so would result in an extrapolation of the data. For if several different fixed rations had been fed to each lot of steers instead of only a single ration, then it is possible that the gain isoquants would have a different shape or curvature than the gain isoquants where each lot of steers were fed only a single fixed ration throughout the feeding period. The difference in the shape of the isoquants would indicate that there is some physiological differences in feeding a single fixed ration as compared to feeding several different fixed rations during the feeding period.

The basis for these statements may be illustrated in Figure 13, where OPE is the expansion path for a given feed price ratio. A single fixed ration OR_k may be fed to produce 300 pounds of beef gain and the beef animal will be of grade A as indicated by the iso-grade curve A. This fixed ration, OR_k, will be the least cost ration if only a single ration is to be fed during the feeding period. If 200 pounds of gain and grade B beef are to be produced, then ration OR_k is no longer the least-cost ration. The least-cost ration for 200 pounds gain is given by feeding ration OR_j.

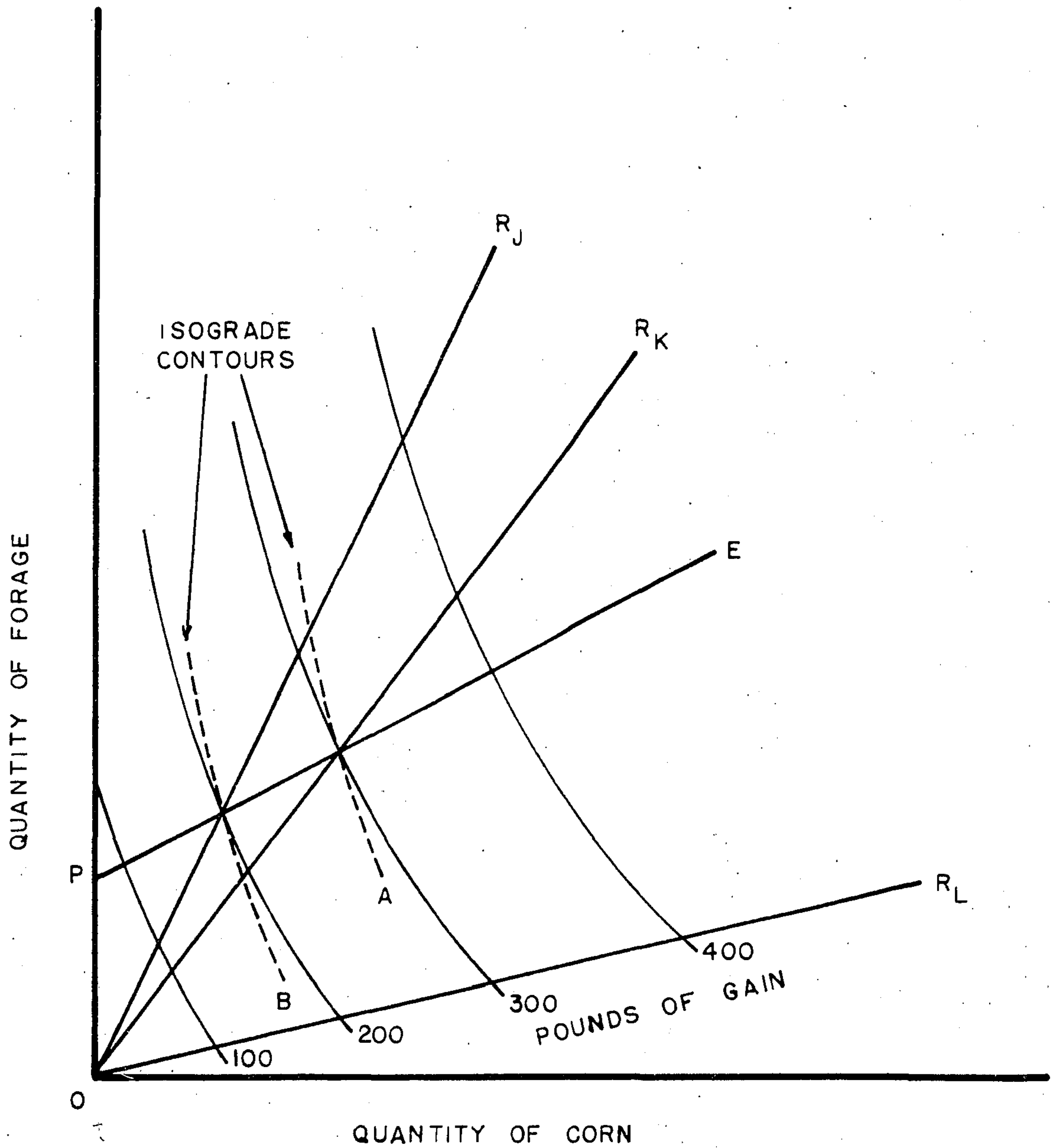

Figure 13. An isoquant map showing least-cost rations for specified levels of gain

Time Functions

The production surface, discussed in the previous sections, indicates how any given level of gain may be attained with various combinations of the two feeds -- corn and forage. However, the time required for a beef animal to take on gains will vary depending upon the particular ration that was fed. Therefore, the amount of gain that will be produced in any given feeding period will depend upon the ration that has been fed. The ration that produces the fastest gains need not necessarily coincide with the least-cost ration in terms of gain. Hence, the most profitable feeding program may be a feeding program where a ration, other than the least-cost ration, is fed in order to place the cattle on the market at a time when the market price is such as to more than compensate for the added cost of feeding a ration that results in faster gains than the least-cost ration.

While a ration of all forage will produce gains, the gains from such a ration will be less per pound of feed than the gains from rations containing successively greater proportions of concentrates. The time required to consume a given quantity of feed of an all forage or bulky ration is generally considered to be greater than the time required to consume the same quantity of feed of a ration that contains some concentrate and is less bulky. This difference in feed intake per unit of time is, at least, partially due to the bulk handling capacity of the intestinal tract (48, p. 6), which throughout the remainder of the thesis will be referred to as stomach capacity.* Thus, for feeder cattle that are on full feed, the stomach capacity of the feeder cattle will determine the quantity of feed of each ration that will be consumed for any given period of time.

The basis of these statements can be explained by the use of Figure 10. Given a feeding period of time t, then a beef animal could consume the quantity OF of the all forage ration to produce a gain FG. Or if the beef animal was fed along the OR_L ration line for the same length of time, t, to OC, then CG' gains would be produced, where $CG' > FG$.

With a beef animal fed for a given period of time, t, on an all forage ration, then OF represents the stomach capacity of that animal for the given time t. Now if forage is replaced by successive quantities of corn then this replacement ratio would take place along FC, in Figure 10. Therefore, the curve FC would depict the limit of the stomach capacity for the various rations for a feeding period of time t.

The counterpart of the stomach capacity curve, FC, is the curve GG' which denotes the maximum amount of gains pos-

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*Strictly speaking, the feed intake of beef cattle is limited by the 1) bulk handling capacity of the intestinal tract and by the 2) daily intake of total digestible nutrients (48, p. 6).

sible from the various rations fed for a given time t, and may be called the "limiting gain curve". As corn replaces forage along FC, gains increase until they are a maximum at CG'.

The FC curve in the feed plane is also an isotime curve in that it represents the various quantities of different rations that may be consumed in a given length of time. Thus, FC may be looked upon as an isotime curve or the curve denoting the stomach capacity of the beef animal. There exists a family of isotime curves in the feed plane. Each isotime curve denotes a different length of feeding time and for each isotime curve there corresponds a different "limiting gain curve" on the production surface denoting maximum gains for each ration for the relevant feeding time.

The production surface OGG' in Figure 10 represents the gains that may be attained from various combinations of forage and corn within the limits of 1) the physiologically feasible corn-forage rations and 2) the maximum quantities of the various corn-forage rations that may be fed within time t. That is, the production surface, OGG', is limited by the limiting ration OR_L and the stomach capacity curve FC for the given feeding time t.

The two-dimensional representation of the production surface is shown in Figure 12. The curve OR_L represents the limiting corn-forage ration. The FC curve is the stomach capacity or isotime curve which corresponds to the GG' curve on the production surface which is the "limiting gain curve". F'C' denotes a different stomach capacity curve for a different length of feeding time.

Grade Functions

Even though different rations can be fed to beef cattle to produce gains, the length of the feeding period required to produce a given level of gain will, however, differ for each ration. That is, the rate at which feeder cattle gain weight will depend, _ceteris paribus_, on the ration fed. The rate of gain will generally increase with the level of corn in the ration. Thus, the total amount of gain attained for a specified feeding period will depend upon the ration fed. If cattle of the same grade are divided into separate lots so that they may be fed different rations of corn and forage, then generally they will not put on gains at the same rate and hence, will not be of the same grade at the end of a given feeding period.

The price for which the cattle may be sold at the end of a given feeding period depends, _ceteris paribus_, upon their grade. The price for the various grades will, however, vary during the feeding period due to seasonal price changes.*

*Cyclical price changes as well as random price fluctuations will influence the price for the various grades of beef cattle.

Therefore, the price at any one time for which the cattle may be sold will depend upon the grade of the cattle and the price for that particular grade at the time of sale.

Since beef grades are measured in subjective terms, it is necessary to code these terms in order that a numerical analysis may be made. One method of coding is to use the market prices of the various grades. The coding of the subjective grade terms will allow for a grade function which expresses the beef grades as a function of the corn and forage fed.

A grade surface may be constructed which is similar, but not identical, to the production surface in Figure 10 and where the isoquant curves are modified and replaced by isograde curves. The isograde curves represent all possible combinations of forage and corn that produce a given grade of beef. The isograde curves for a grade surface may be illustrated in a two-dimensional figure in the same manner as isoquants for a production surface are illustrated. Such a two-dimensional isograde map is shown in Figure 14, where OR_L is the physiological limiting ration that may be fed and g_1, g_2, $\cdots$, g_5 are isograde curves that indicate the various combinations of corn and forage that may be fed to produce a given grade of beef.

While various rations may be fed to produce a given level of gain, the gain isoquant does not represent one grade

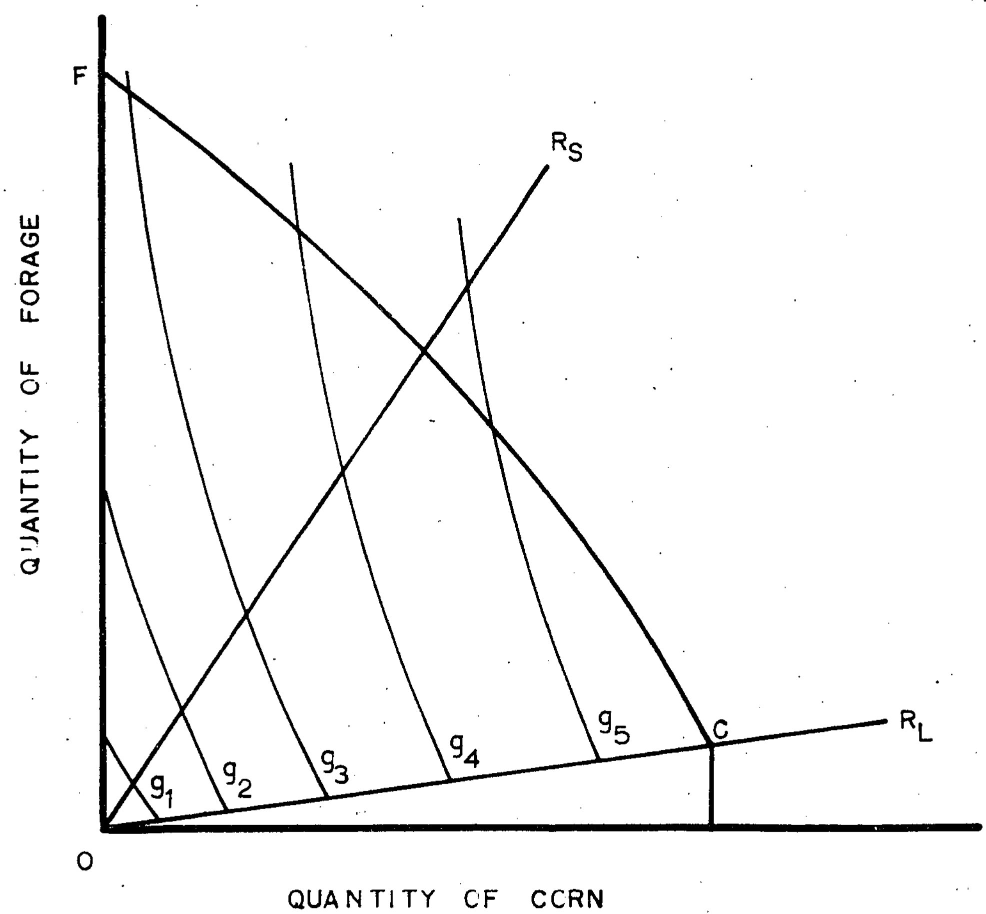

Figure 14. Isograde map for beef cattle

of beef but rather several grades depending upon the ration fed. Therefore, the gain isoquants in Figure 12 cannot serve as isograde contours and, hence, any one isoquant would represent various grades of beef.

A beef animal fed the OR_L ration will attain a given grade, say g_4, in less time than a beef animal fed the OR_s ration and furthermore, the two animals will generally have taken on different amounts of gain in order to attain the same grade. An increase in the value of a beef animal fed on the all forage ration is due primarily to the increase in weight with little, if any, change in grade, while the change in value of the beef animal fed the OR_L ration is due primarily to the change in grade which is associated with the rate of gain.

Since different rations produce different grades (quality) of beef and since the rate of gain is different for different rations, then for a given length of feeding time there would exist some ration that would produce the maximum value of beef. If FC, in Figure 14, is an isotime curve, identical to FC in Figure 12, then ration OR_L is the ration that produces the maximum value of beef for the given feeding time denoted by the FC curve.

The isograde map, in Figure 14, also shows that an all forage ration or rations with a high proportion of forage will not produce top quality beef. For rations that are made up primarily of forage do not furnish an adequate supply of total digestible nutrients for the fattening process. Experimental results have shown that concentrate to roughage ratios between 30:70 and 70:30 produce satisfactory liveweight gains in fattening cattle (48, p. 6). Even though ration OR_L may produce the maximum value of beef for a given feeding period, this ration may not be the least-cost ration in terms of gain or the ration that will maximize profits. In order to determine the optimum ration, feed costs need to be considered.

Optimum Feeding Program

Paradoxically, the least-cost ration in terms of gain may not necessarily be the ration that results in maximum profits. This paradox arises because different rations produce different grades (quality) of beef which sell for different prices. Furthermore, as cattle feeders well know, the time at which the cattle are marketed may affect profits as much or more than the cost of feed.

In a beef fattening program, the time of marketing and the grade produced are influenced by the quantity and quality of the feed intake. For example, a high corn ration will produce cattle of a given grade in less time than a high forage ration. However, the cost will be greater for the high corn ration than for the high forage ration. Nevertheless, the higher cost ration may be the most profitable ration to feed depending upon the trend of market prices for finished cattle. If the high corn ration is the most profitable ration, then in a cattle feeding program corn will substitute not only for forage but will also substitute for time. Therefore, the most profitable feeding operation may be to feed a ration that does not necessarily minimize feed costs in terms of gain but rather increases the grade of the cattle so that they may be finished for the market ahead of any decline in the seasonal price.

The optimum feeding program, given the original cost of the feeder cattle, will depend upon 1) the price of the feed inputs, 2) the time required to put on a given amount of gain (*i.e.*, the rate of gain), 3) the grade of the cattle at the end of the feeding period and 4) the price of the various grades at the time of marketing.

EMPIRICAL RESULTS

A beef feeding experiment extending over three years, 1957, 1958 and 1959, serves as the basis for this study. This experiment was conducted cooperatively by the Department of Animal Husbandry, the Department of Agronomy and the Department of Economics and Sociology in order to obtain data to estimate the feed relationships for fattening beef steers fed on soilage (fresh-chopped pasture forage) and corn.

Experimental Design

The beef feeding experiment, which was designed (31, 32, 33, 34, 35) to determine the feed relationships of soilage and corn for fattening beef steers, was conducted at two experimental farms in Iowa, the Western Iowa Experimental Farm at Castana and the Soil Conservation Experimental Farm at Shenandoah. The only difference in the experimental design between the two experimental farms was in the rations. The rations at the Western Experimental Farm contained stilbestrol while the rations at the Soil Conservation Farm did not. The soilage (*i.e.*, fresh-chopped pasture forage) and corn that was fed to the cattle at both experimental farms was mixed and fed in fixed proportions. Also, the cattle were full fed in that they were fed all of a given ration (*i.e.*, a fixed proportion of soilage and corn) that they would clean up with a minimum amount of wastage.

Experimental cattle

The cattle used in the experiment, for any one year, were Hereford steers purchased the preceding fall on the Omaha market as choice feeders. After the steers were purchased they were divided between the two experimental farms and wintered on a wintering ration to gain about one pound per day. The following spring, about a week before the beginning of the soilage feeding experiment, half the steers wintered at each farm were then transferred to the other farm. The steers were then allowed access to pasture for a conditioning period prior to the beginning of the soilage feeding experiment.

At the start of the soilage feeding experiment the steers were individually weighed on two successive days and then on the basis of the average of these two weights, the wintering location and the winter gains, they were placed in eight lots of seven steers each. Four of the steers in each lot had been wintered at the Castana Farm and three at the Shenandoah Farm. The experimental treatments were then randomly assigned to the eight lots of steers. The steers were individually weighed at definite intervals throughout the course of the experiment. In 1957 the steers were individually weighed at 28 day intervals while in 1958 and 1959 they were weighed at

21 day intervals. The steers were also individually weighed on two successive days at the end of the soilage feeding experiment, as at the beginning, and at any time the cattle were sold. In all cases the average of the two weights was used as the weight for that particular time.

The length of time the cattle were fed differed between years and also between the experimental farms. At the Castana Farm the cattle were fed for 133 days in 1957, 144 days in 1958 and for 132 days in 1959 for an average feeding period of 136 days. At the Shenandoah Farm the cattle were fed for 138 days in 1957, 144 days in 1958 and for 132 days in 1959 for an average feeding period of 138 days.

The steers were appraised at definite intervals during the experiment and whenever the average grade of any one lot was appraised as low choice they were sold. Similar lots at both farms were sold at the same time.

The steers, at the beginning of the soilage feeding experiment, were good-to-choice feeder steers weighing approximately 850 pounds. For any one year 56 head of steers were required for the soilage feeding experiment at each farm or a total of 112 head for both farms. Thus, the results of this experiment, covering a period of three years, are based on the performance of a total of 336 steers.

Experimental treatments

The soilage feeding experiment at each farm for each year was made up of eight lots of steers with seven steers per lot, which were randomly assigned the eight treatments. The treatments in this experiment are the different rations of soilage and corn. For any one year at each farm there were six different treatments and two replicated treatments for a total of eight treatments. Two of the six different treatments were duplicated each year such that after three years all treatments were included in the experiment the same number of times. The treatments were identical at each experimental farm except stilbestrol was included in the treatments at Castana and not at Shenandoah. Table 1 shows the number and kind of experimental treatments and which treatments were replicated each year for the three year period.

Feed supply

Six feed combinations or rations were fed in this experiment ranging from all soilage to 2 parts soilage and 1 part corn. Due to the difficulty of determining the amount of forage consumed by an animal when on pasture, the pasture forage was fed as soilage. The pasture forage was an alfalfa-bromegrass mixture and predominantly alfalfa with bromegrass making its main contribution during the first clipping. The soilage was chopped once daily and was fed fresh along with

Table 1. The experimental treatments or rations[a] for the 3 year period - 1957, 1958 and 1959

Lot number	Year		
	1957	1958	1959
1	All soilage	All soilage	All soilage
2	20:1	20:1	20:1
3	10:1	10:1	10:1
4	5:1	5:1	5:1
5	3:1	3:1	3:1
6	2:1	2:1	2:1
7	All soilage	20:1	10:1
8	5:1	3:1	2:1

[a]The ration is the ratio of soilage to corn.

the proper amount of concentrate and supplement.

The concentrate fed in this experiment was ground shelled corn, about 14 percent moisture. The supplement fed to the cattle at the Castana farm included stilbestrol, whereas, the supplement fed at the Shenandoah farm did not include stilbestrol. The supplement fed at each location was fed to all lots at the rate of 1 pound per head per day throughout the experiment. At this feeding rate the cattle at the Castana farm were to be fed 10 milligrams of stilbestrol daily. For the all soilage ration, alfalfa meal was used as the main constituent in the supplement while ground shelled corn was the main constituent in the supplement for all other rations. Table 2 shows the composition of the supplement fed at the two farms.

Estimation of the Production Function

On the basis of economic logic and animal nutrition theory several different algebraic equations were fit to the data obtained from the experiment in an attempt to predict

Table 2. Composition of the supplement fed at Castana and Shenandoah

	Castana		Shenandoah	
	All soilage lots (lbs.)	Lots receiving corn and soilage (lbs.)	All soilage lots (lbs.)	Lots receiving corn and soilage (lbs.)
Alfalfa meal	80.0	--	80.0	--
Ground corn	--	80.0	--	80.0
Dried molasses	10.0	10.0	10.0	10.0
Dicalcium phosphate	6.0	6.0	6.0	6.0
Salt	2.5	2.5	3.5	3.5
Trace mineral premix	0.5	0.5	0.5	0.5
Stilbestrol premix	1.0	1.0	--	--
Total	100.0	100.0	100.0	100.0

the beef cattle production surface. Each alternative algebraic function was fit separately to the data from each farm. These functions have been denoted the overall* functions either with or without stilbestrol. Having once obtained the two overall functions, a comparison of the feed-gain relationships can be made between different rations either with or without stilbestrol. Interest is mainly in the overall functions for they more closely represent the environment within which a farmer must make his decisions. Since a farmer is unable to predict the outcome for any individual year, he must make decisions in an environment of uncertainty. The overall functions thus represent an "expected" or an average outcome upon which decisions may be based.

The overall functions express total gain from the beginning of the experiment as a function of the total feed consumed since the beginning of the experiment and the deviations of the average maximum temperature of each observation interval from the mean maximum temperature for the overall feeding period.

Experimental observations were taken at definite intervals throughout the course of the experiment on the amount of gain and the consumption of the different feeds. For the all soilage rations, the alfalfa meal that was fed in the supple-

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*"Overall" refers to the combined feeding periods of all 3 years at any one farm.

ment was converted to a soilage basis* and then combined with the soilage fed to give a total soilage (forage) input. For all other rations, the corn that was fed in the supplement was combined with the corn that was fed in the rations to give a total corn input. The alfalfa meal and the corn that was fed in the supplement has been combined with the soilage and corn that was fed in the ration in order to get better estimates of the substitution rates between the two feeds. Table 3 shows the composition of the supplement after the corn and alfalfa meal has been removed. The supplement as shown in Table 3 would be fed at the rate of .2 of a pound per head per day.

After the gain and feed observations had been put on a per steer basis they were cumulated progressively to give a cumulative series of gains, soilage consumption and corn consumption per steer from the beginning of the feeding experiment. The daily maximum temperature** for each feed-gain

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*Alfalfa meal was converted to soilage by the following method:

$$\text{lbs. of soilage} = (\text{lbs. of alfalfa meal}) \frac{(\%\ \text{dry matter of alfalfa meal})}{(\%\ \text{dry matter of soilage})}.$$

The per cent dry matter of good alfalfa meal was obtained from Morrison's feeding tables (46, p. 1086) and the per cent dry matter of soilage was obtained by taking the mean per cent dry matter from samples of the soilage that was fed.

**Climatological data for the Western Iowa and the Soil Conservation Experimental farms are available through U. S. Department of Commerce Climatological reports (57).

Table 3. Composition of the supplement for the stilbestrol and the non-stilbestrol rations

	Feeder steers receiving stilbestrol (lbs.)	Feeder steers not receiving stilbestrol (lbs.)
Dried molasses	50.0	50.0
Dicalcium phosphate	30.0	30.0
Salt	12.5	17.5
Trace mineral premix	2.5	2.5
Stilbestrol premix	5.0	--
Total	100.0	100.0

observation interval was listed and then for each interval the temperature data were averaged to give an average maximum temperature for each feed-gain observation interval. A temperature series was then obtained by taking the deviations of the average maximum temperature for each feed-gain observation interval from the mean maximum temperature for the overall feeding period. This series was then used along with the cumulative series of gain, soilage consumption and corn consumption to estimate the production surface.

<u>Autocorrelation</u>

In estimating the coefficients in the production function when the observations are not independent gives rise to problems of autocorrelation. While feed-gain observations between lots of steers are independent, successive observations on any one lot of steers are not independent. In order for feed-gain observations on any one ration to be independent would require as many lots of steers that were fed the same ration as there were observations so that each lot would be observed one time only.

If the coefficients of the production function were estimated by least squares under the assumption that the error terms, u_t (where t is an index of time), have the following properties:

(12)

(a) The errors, u_t, are uncorrelated with each of the independent variables in the equation.

(b) $E(u_t) = 0$, and the u_t's are normally distributed.

(c) $E(u_t^2) = \sigma^2 < \infty$

(d) $E(u_t u_s) = 0 \qquad t \neq s$

then the coefficient estimates are the best linear unbiased estimates. However, if there is autocorrelation in the errors, u_t, and they follow the autoregressive scheme:

$$u_t = \beta u_{t-1} + e_t$$

where β is the autocorrelation coefficient and e_t is a random variable with the following properties:

(13)
(a) The errors, e_t, are uncorrelated with each of the independent variables in the equation.
(b) $E(e_t) = 0$, and the e_t's are normally distributed.
(c) $E(e_t^2) = \sigma^2 \quad < \infty$
(d) $E(e_t e_s) = 0 \qquad t \neq s$

and if the production function is estimated under the assumptions given by the equations in 12 when the errors are really autocorrelated, then the estimates remain unbiased and consistent but are no longer efficient (12, p. 51).

While the presence of autocorrelation in the estimating equation does not bias the regression coefficients or make them inconsistent it does, however, affect their variances and covariances (58, p. 170). Wold (62, p. 44) states that if the residuals are not autocorrelated, then the coefficients estimated by least squares are unbiased and the usual statistical test of the coefficients is valid. If, however, the residuals are autocorrelated, then the question of significance is "... subject to a considerable margin of indeterminancy".

Cochrane and Orcutt (12, pp. 48-49) show that the method of least squares when applied in the usual manner to relationships that contain "... highly positively autocorrelated error terms results in an extremely inefficient use of data ...." Furthermore, they point out that most of the efficiency may be regained by a transformation that will make the error terms approximately normal.

To make tests of significance and to construct confidence limits it is necessary that the error terms be random. If the error terms, that were highly autocorrelated, have been made random by a transformation then it is possible to make tests of significance and construct confidence limits in the usual manner (12, p. 51).

Basic equations

One of the equations used to estimate the production surface was a quadratic function* of the type:

$$(14) \quad G_t = a_1C_t + a_2F_t + a_3C_t^2 + a_4F_t^2 + a_5CF_t + u_t$$

where G refers to pounds of beef gain, C refers to pounds of corn, F refers to pounds of soilage, the a_i's ($i = 1, \ldots, 5$) are constants to be estimated, u is a random variable and t is an index of time. The quadratic production function is estimated without a constant term under the assumption that when corn and forage intake is zero beef gains will also be zero.

---

*Previous work with the data indicated that the quadratic function consistently gave a better statistical fit than did the linear, Cobb-Douglas, or the square root functions. The results of a modified Cobb-Douglas function and an exponential function are reported in Appendix A.

In order to remove the effects of autocorrelation, as discussed in the previous section, the assumption was made that the random variable u_t was generated by the autoregressive scheme

$$(15) \qquad u_t = \beta u_{t-1} + a_6 H_t + e_t$$

where β is the autocorrelation coefficient, H is a temperature variable, a_6 is a constant to be estimated and e_t is a random variable with the properties given by the equations in 13.

The temperature variable is included in equation 15 under the assumption that temperature would increase or decrease beef gains depending upon the temperature for each observation interval.

Equation 14 can be written for t-1 as

$$(16) \quad G_{t-1} = a_1 C_{t-1} + a_2 F_{t-1} + a_3 C^2_{t-1} + a_4 F^2_{t-1} + a_5 CF_{t-1} + u_{t-1} \, .$$

Now equation 16 can be solved for u_{t-1} and substituted into equation 15 to give

$$(17) \quad u_t = \beta (G_{t-1} - a_1 C_{t-1} - a_2 F_{t-1} - a_3 C^2_{t-1} - a_4 F^2_{t-1} - a_5 CF_{t-1}) + a_6 H_t + e_t \, .$$

If equation 17 is now substituted into equation 14, the following equation is obtained

$$(18)\quad (G_t - \beta G_{t-1}) = a_1(C_t - \beta C_{t-1}) + a_2(F_t - \beta F_{t-1}) + a_3(C_t^2 - \beta C_{t-1}^2) + a_4(F_t^2 - \beta F_{t-1}^2) + a_5(CF_t - \beta CF_{t-1}) + a_6H_t + e_t \ .$$

If the variables in equation 14 are now replaced by the transformed variables in equation 18, then the errors, e_t, are not autocorrelated and the least squares method of estimation will apply (55, p. 324). In order to make such a transformation requires some knowledge of the autocorrelation coefficient β. An empirical estimate of the autocorrelation coefficient was made independent of the functional form used to estimate the production surface. This estimate was obtained from the gain observations by taking the deviations from the observation period means of the replicated lots and then regressing the deviations for observation period t on the deviations for observation period t-1. This estimate of β is a maximum likelihood estimate.* This same procedure was carried out for all 3 years at both stations to obtain an average autocorrelation coefficient. The autocorrelation coefficient, β, estimated by this procedure turned out to .89541808 with a standard error of .07090565. This coefficient is highly significant at the .1 per cent probability

*The author is indebted to Dr. Wayne A. Fuller for the proof that this estimate of β is a maximum likelihood estimate. The proof is given in Appendix B.

level.*

Using this estimate of the autocorrelation coefficient the variables in equation 14 were then transformed as indicated in equation 18. The transformed variables were then used to obtain least square estimates of the coefficients in the production function.

The production functions estimated using the quadratic function are:**

I. The overall stilbestrol function

$$(19) \quad G = .116371500C + .02316051F - .000004995 5C^2 - .0000007455F^2 + .0000000374CF - 1.2236046H$$

II. The overall non-stilbestrol function

$$(20) \quad G = .14971812C + .02128774F - .0000122612C^2 - .0000005775F^2 - .0000037907CF - 2.2005042H.$$

The coefficient of determination, standard errors and the "t" values for the overall stilbestrol and non-stilbestrol production functions are presented in Tables 4 and 5, respectively. The coefficient of determination is quite high for both the stilbestrol and the non-stilbestrol functions

---

*The "t" value for the estimated coefficient is 12.6283 with 143 degrees of freedom.

**In addition to the two overall production functions an "aggregate" production function has also been computed. This aggregate function is obtained by fitting the quadratic function collectively to both the stilbestrol and non-stilbestrol data. The aggregate production function along with the isoquant schedules and the marginal rates of substitution is presented in Appendix C.

Table 4. Coefficient of determination, standard errors and "t" values for the overall stilbestrol production function (equation 19)

R^2	Independent variable	Standard error of regression coefficient	"t" value	Level of significance
.9784	C	.01626541	7.155	$p < .001$
	F	.00231212	10.017	$p < .001$
	C^2	.00000584	.855	$.20 < p < .40$
	F^2	.00000018	4.202	$p < .001$
	CF	.00000202	.019	$p > .50$
	H	.30797286	3.973	$p < .005$

Table 5. Coefficient of determination, standard errors and "t" values for the overall non-stilbestrol production function (equation 20)

R^2	Independent variable	Standard error or regression coefficient	"t" value	Level of significance
.9718	C	.01594771	9.388	$p < .001$
	F	.00216274	9.843	$p < .001$
	C^2	.00000571	2.148	$.01 < p < .05$
	F^2	.00000014	4.197	$p < .001$
	CF	.00000166	2.288	$.01 < p < .05$
	H	.30616731	7.187	$p < .001$

indicating that the quadratic function explains a major portion of the variance in beef gains. All of the variables in the non-stilbestrol function are significant at least at the 5 per cent probability level whereas, certain variables in the stilbestrol function are acceptable only at a very low level of probability. Nevertheless, these variables have been retained in the production function since they appear to be consistent with nutrition and production logic.

The coefficient on the temperature variable (H) for both the stilbestrol and non-stilbestrol function is significant at least at the .5 per cent level of probability. The negative sign on the temperature coefficient indicates that as temperature rises gains will be less. This result is consistent with nutrition theory.

Gain Isoquants and Substitution Rates

Once the production function has been determined equations for isoquants, isoclines and factor-product transformation relationships may be derived. The beef gain isoquant equations for stilbestrol and non-stilbestrol rations can be derived from the two overall production function equations 19 and 20, respectively. The beef gain isoquant equations are:

I. With stilbestrol

$$(21)\quad F = 15,533.54124 + .02508380C \pm (-670,690.811)\Big[(.02316051 + .0000000374C)^2 + .00000298\,(.11637150C - .0000049955C^2 - 1.2236046H - G)\Big]^{1/2}$$

II. Without stilbestrol

$$(22)\quad F = 18,430.94372 - 3.28199134C \pm (-868,800.865)\Big[(.02128774 - .00000379070C)^2 + .00000231\,(.14971812C - .0000122612C^2 - 2.2005042H - G)\Big]^{1/2}.$$

The isoquant equations express soilage (F) as a function of corn (C), the level of gain (G) and temperature (H).* If beef gains are held constant at a given level, then the isoquant equations will specify all possible combinations of soilage and corn that will produce this given level of gain.

Equations for determining the marginal rates of substitution between soilage and corn for the stilbestrol and non-stilbestrol rations can be derived from the isoquant equations 21 and 22, respectively. The equations for predicting the marginal rates of substitution of corn for soilage are:

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*While the temperature variable is included in the isoquant equations, as it will be in all other equations, the temperature will be fixed at the overall mean for most of the analysis which follows, unless otherwise stated. The overall mean temperature for the stilbestrol feeding period was 79.36 degrees Fahrenheit while the overall mean temperature for the non-stilbestrol feeding period was 83.69 degrees Fahrenheit.

I. With stilbestrol

$$(23) \quad \frac{\partial F}{\partial C} = \frac{.11637150 + .0000000374F - .000009991C}{.02316051 + .0000000374C - .000001491F}$$

II. Without stilbestrol

$$(24) \quad \frac{\partial F}{\partial C} = \frac{.14971812 - .0000037907F - .0000245224C}{.02128774 - .0000037907C - .000001155F}$$

While the above equations define the marginal rate of substitution of corn for soilage, the marginal rate of substitution of soilage for corn can be derived as the reciprocal of the above equations.

Beef gain isoquant schedules and the marginal rates of substitution associated with them have been derived for 100, 200, 300, 350 and 400 pounds of beef gain. Beef gain isoquant schedules along with the associated marginal rates of substitution are presented in Tables 6 and 7 for the stilbestrol and non-stilbestrol functions, respectively. Similarly, gain isoquants corresponding to the data in Tables 6 and 7 are presented in Figures 15 and 16.

The experiment upon which this study is based did not include rations of soilage to corn beyond the 2:1 ration. Therefore, the extension of the isoquant schedules beyond the 2:1 ration results in an extrapolation of the data.

The rate at which corn will substitute for soilage in any one beef fattening ration for a given level of gain is indicated by the slope at a particular point on the isoquant. The rate of substitution indicates, for any one level of gain,

Table 6. Isoquant schedules, derived from the overall stilbestrol quad: combinations[a] and marginal rates of substitution of corn for : 850 pound good-to-choice feeder steers (temperature held cons

Lbs. corn	100 lbs. gain Lbs. soilage	Ration[c]	$\frac{\partial F}{\partial C}$[b]	200 lbs. gain Lbs. soilage	Ration	$\frac{\partial F}{\partial C}$	300 Lbs. soilage
0	5,182	--[d]	7.55				
100	4,456	44.56	6.99				
200	3,780	18.90	6.53	13,506[e]	67.53	37.88	
300	3,147	10.49	6.14	11,127	37.09	17.29	
400	2,550	6.38	5.81	9,652	24.13	12.83	
500	1,984	3.97	5.51	8,490	16.98	10.62	
600	1,447	2.41	5.25	7,503	12.50	9.22	
700	933	1.33[e]	5.01	6,632	9.47	8.24	
800				5,846	7.31	7.50	
900				5,126	5.70	6.92	
1,000				4,459	4.46	6.44	
1,100				3,836	3.49	6.04	13,732[e]
1,200				3,250	2.71	5.69	11,382[e]
1,300				2,696	2.07	5.39	9,954[e]
1,400				2,170	1.55[f]	5.13	8,832[e]
1,500							7,882
1,600							7,045
1,700							6,291
1,800							5,601
1,900							4,962
2,000							4,366
2,100							e,806
2,200							
2,300							
2,400							
2,500							
2,600							
2,700							
2,800							
2,900							

[a]For each of the feed combinations there would also be fed a certai be fed at the rate of .2 of a pound per day. The estimated number of fe in Table 36.

[b]The marginal rate of substitution of corn for soilage.

[c]Ration is the ratio of soilage to corn.

[d]The all soilage ration.

[e]The estimated feeding period exceeds the 136 day average feeding p

[f]All feed combinations at this point exceed the 2:1 ration and, hen

quadratic function, showing possible feed
for soilage at five gain levels, for
constant at the overall mean.

300 lbs. gain			350 lbs. gain			400 lbs. gain		
. age	Ration	$\frac{\partial F}{\partial C}$	Lbs. soilage	Ration	$\frac{\partial F}{\partial C}$	Lbs. soilage	Ration	$\frac{\partial F}{\partial C}$
32[e]	12.48	38.81						
32[e]	9.48	16.81						
54[e]	7.66	12.40						
32[e]	6.31	10.23						
32	5.25	8.87						
5	4.40	7.91	13,192[e]	8.24	28.40			
1	3.70	7.19	11,201[e]	6.59	15.30			
01	3.11	6.63	9,879[e]	5.49	11.62			
52	2.61	6.16	8,822[e]	4.64	9.69			
56	2.18	5.77	7,918[e]	3.96	8.46			
06	1.81[f]	5.44	7,118	3.39	7.58	13,431[e]	6.40	29.83
			6,396	2.91	6.90	11,412[e]	5.19	15.22
			5,733	2.49	6.37	10,103[e]	4.39	11.46
			5,119	2.13	5.93	9,063[e]	3.78	9.52
			4,545	1.82[f]	5.56	8,176[e]	3.27	8.29
						7,393[e]	2.84	7.41
						6,687[e]	2.48	6.74
						6,040[e]	2.16	6.21
						5,441	1.88[f]	5.78

rtain amount of the supplement shown in Table 3. This supplement would
f feeding days for each of the feed combinations in this table are shown

ng period in the experiment. See Table 36.

hence, are outside the limits of this experiment.

Table 7. Isoquant schedules, derived from the overall non-stilbestrol q feed combinations[a] and marginal rates of substitution of corn i for 650 pound good-to-choice feeder steers (temperature held c

	100 lbs. gain			200 lbs. gain			300
Lbs. corn	Lbs. soilage	Ration[c]	$\frac{\partial F}{\partial C}$[b]	Lbs. soilage	Ration	$\frac{\partial F}{\partial C}$	Lbs. soilage
0	5,526	--[d]	8.64	17,773[e]	--	142.53	
100	4,677	46.77	8.35	15,441[e]	154.41	28.88	
200	3,854	19.27	8.10	13,236	66.18	18.06	
300	3,056	10.19	7.87	11,623	38.74	14.62	
400	2,280	5.70	7.66	10,262	25.66	12.76	
500	1,524	3.05	7.47	9,051	18.10	11.54	
600	786	1.31[f]	7.29	7,943	13.24	10.66	
700				6,912	9.87	9.99	
800				5,941	7.43	9.44	
900				5,020	5.58	8.99	
1,000				4,140	4.14	8.61	
1,100				3,296	3.00	8.28	
1,200				2,482	2.07	7.99	
1,300				1,696	1.30[f]	7.74	
1,400							
1,500							11,488[e]
1,600							9,350
1,700							7,847
1,800							6,589
1,900							5,474
2,000							4,456
2,100							3,510
2,200							
2,300							
2,400							
2,500							
2,600							

[a]For each of the feed combinations there would also be fed a certain 3. This supplement would be fed at the rate of .2 of a pound per day. Th each of the feed combinations in this table are shown in Table 37.

[b]The marginal rateof substitution of corn for soilage.

[c]Ration is the ratio of soilage to corn.

[d]The all soilage ration.

[e]The estimated feeding period exceeds the 138 day average feeding per

[f]All feed combinations at this point exceed the 2:1 ration and, hence experiment.

on-stilbestrol quadratic function, showing possible
itution of corn for silage at five gain levels,
mperature held constant at the overall mean)

	300 lbs. gain			350 lbs. gain		
F / C	Lbs. silage	Ration	$\frac{\partial F}{\partial C}$	Lbs. silage	Ration	$\frac{\partial F}{\partial C}$
.53						
.88						
.06						
.62						
.76						
.54						
.66						
.99						
.44						
.99						
61						
28						
99						
74						
	11,488[e]	7.66	29.75			
	9,350	5.34	16.97			
	7,947	4.62	13.54			
	6,589	3.66	11.76			
	5,474	2.88	10.61			
	4,456	2.23	9.79			
	3,510	1.67[f]	9.16			
				8,266[e]	3.44	22.53
				6,492	2.60	14.80
				5,161	1.98[f]	12.14

be fed a certain amount of the supplement shown in Table
ound per day. The estimated number of feeding days for
Table 37.

e.

rage feeding period in the experiment. See Table 37.

ation and, hence, are outside the limits of the

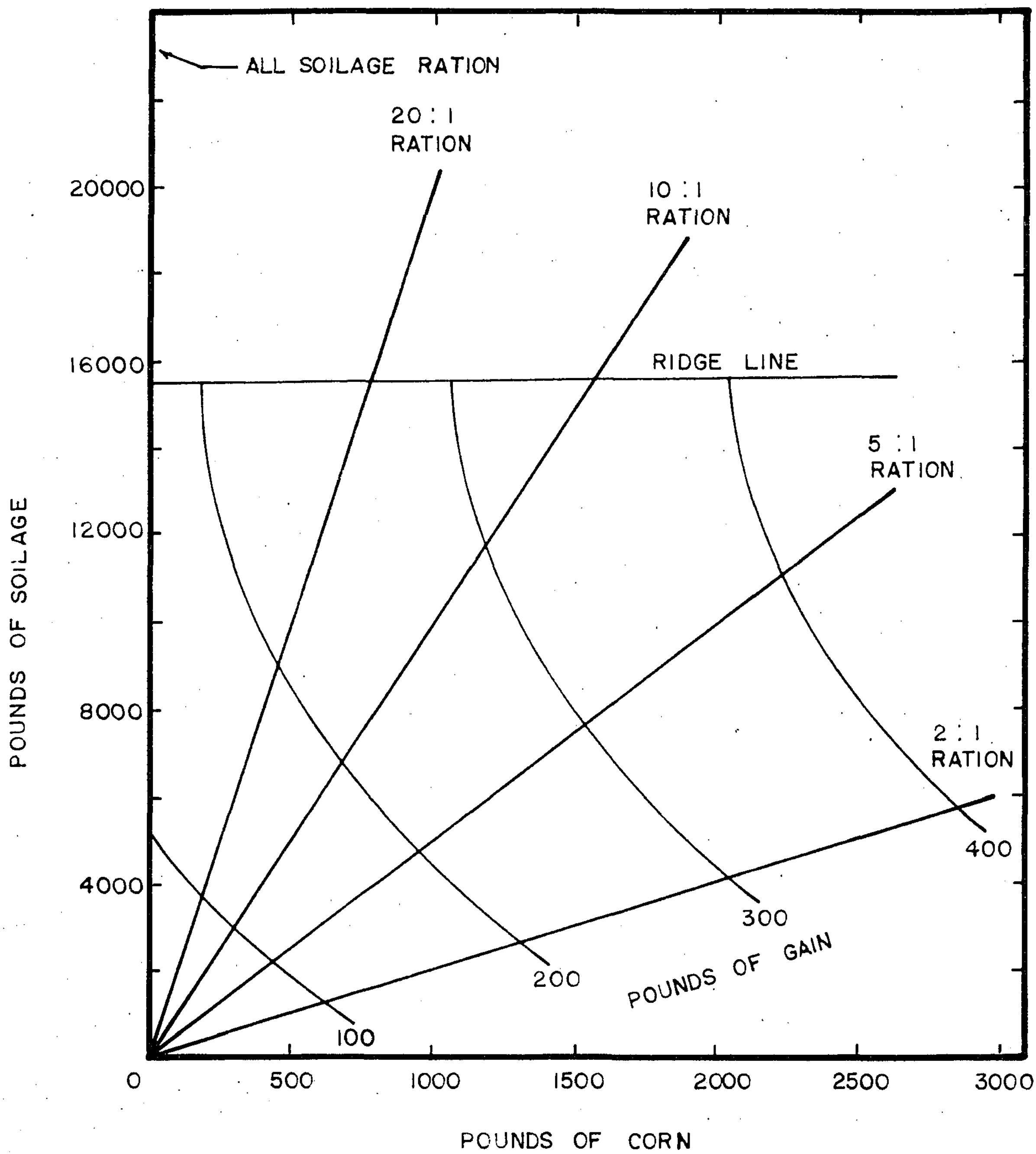

Figure 15. Gain isoquants and selected ration lines for the overall stilbestrol function (temperature held constant at the overall mean)

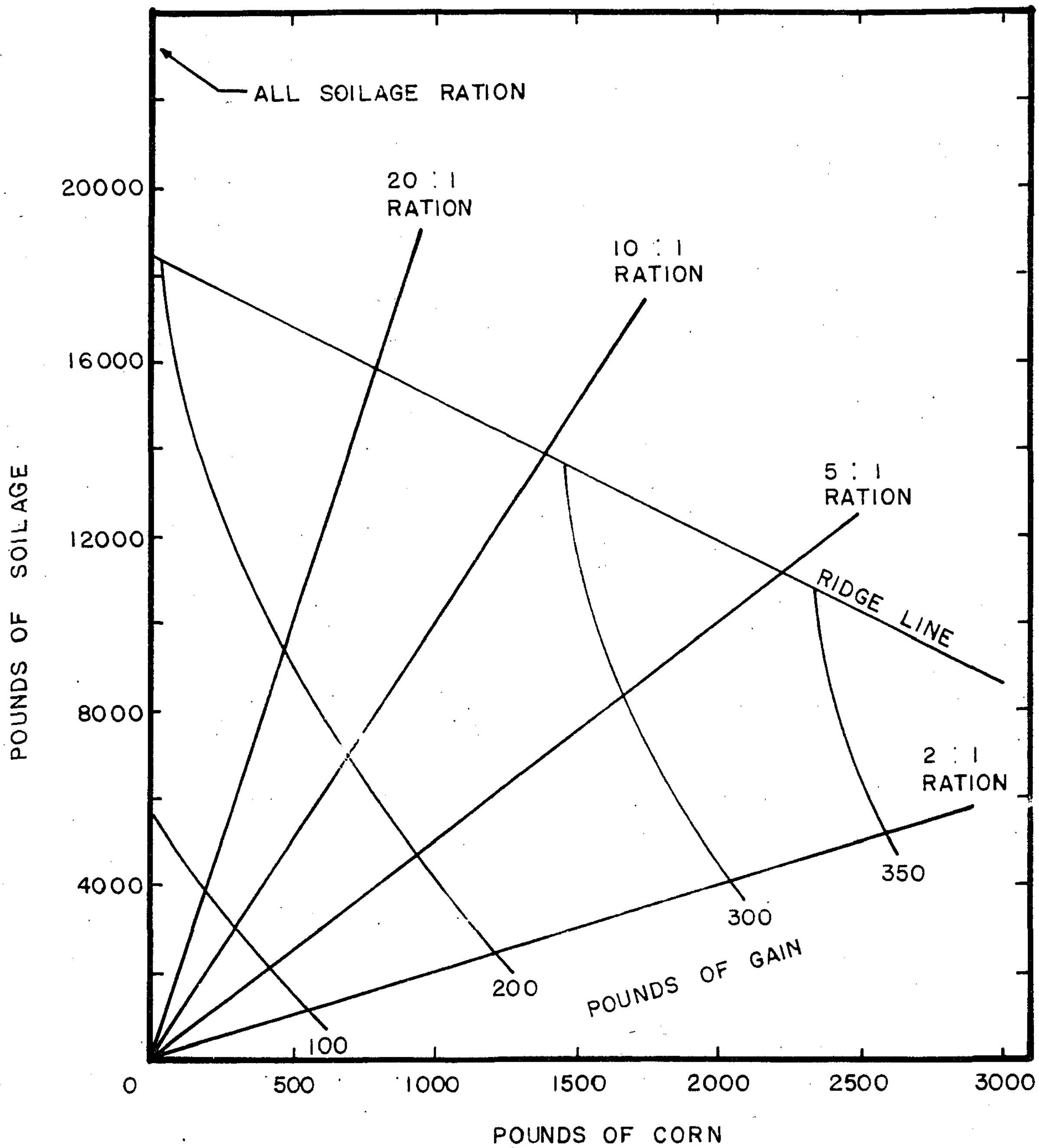

Figure 16. Gain isoquants and selected ration lines for the overall non-stilbestrol function (temperature held constant at the overall mean)

the amount of soilage that may be replaced by a one pound increase in corn. Since the isoquants in Figures 15 and 16 are curved and convex to the origin indicates that the marginal rates of substitution of corn for soilage for all levels of gain are at a diminishing rate. That is, the substitution rates, for any one level of gain, are quite large for rations with a small proportion of corn and then diminish as the proportion of corn in the ration increases. For example, in Table 6, 11,127 pounds of soilage and 300 pounds of corn can be fed in a ration to produce 200 pounds of gain and the rate of substitution of corn for soilage is 17.29. That is, one additional pound of corn will replace 17.29 pounds of soilage. Alternatively, 3,250 pounds of soilage and 1,200 pounds of corn can be fed to produce 200 pounds of gain and the rate of substitution of corn for soilage is now only 5.69.

While Tables 6 and 7 show that the marginal rate of substitution of corn for soilage is at a diminishing rate for all levels of gain, they do not show, at least not very clearly, what the marginal rate of substitution of corn for soilage is along any one ration line at different levels of gain. The rate at which corn and soilage substitute for each other along any one ration line is of interest in cattle feeding for it indicates the relative productivity of the various feeds in the ration as the feeder cattle take on heavier weights.

The prediction equations for estimating the quantities of corn and soilage that are required to produce various levels of gain for different soilage-corn rations, may be derived from the overall production functions and the ration equation:

$$\frac{F}{C} = \alpha \,. \tag{25}$$

The ration equation defines α as the ratio of soilage to corn (*i.e.*, the soilage to corn ratio denotes a given ration α and conversely for a given ration α the quantity of corn in the ration is fixed at a constant proportion of the soilage for all levels of feeding). If equation 25 is rewritten as

$$F = \alpha C \,, \tag{26}$$

then by substituting αC into the production function for F it is possible to derive for various soilage-corn rations the quantities of corn that are required to produce various levels of gain. Once the corn requirements have been determined, for any given ration, the soilage requirements are readily determined from equation 26. However, for the all soilage ration, the isoquant equation can be used directly to determine the quantities of soilage required for various levels of gain.

The derived equations for predicting the quantities of corn that are required to produce various levels of gain for various stilbestrol and non-stilbestrol soilage-corn rations are:

I. With stilbestrol

$$(27)\quad C = -(.11637150 + .02316051\alpha)(-.000001491\alpha^2 + .0000000748\alpha - .000009991)^{-1} \pm (-.000001491\alpha^2 + .0000000748\alpha - .000009991)^{-1}\Big[(.11637150 + .02316051\alpha)^2 - (-.000002982\alpha^2 + .0000001496\alpha - .000019982)(-1.2236046H - G)\Big]^{1/2}$$

II. Without stilbestrol

$$(28)\quad C = -(.14971812 + .02128774\alpha)(-.000001155\alpha^2 - .0000075814\alpha - .0000245224)^{-1} \pm (-.000001155\alpha^2 - .0000075814\alpha - .0000245224)^{-1}\Big[(.14971812 + .02128774\alpha)^2 - (-.00000231\alpha^2 - .0000151628\alpha - .0000490448)(-2.2005042H - G)\Big]^{1/2}.$$

The marginal rates of substitution are estimated for the stilbestrol rations by using equation 23 and for the non-stilbestrol rations equation 24 is used.

The predicted quantities of corn and soilage, for selected rations, at various levels of gain (*i.e.*, 100, 200, 300, 350 and 400 pounds) and the associated marginal rates of substitution of corn for soilage are presented in Table 8 for the stilbestrol rations and in Table 9 for the non-stilbestrol rations. The ration lines corresponding to the data in Tables 8 and 9 have been plotted, respectively, in Figures 15 and 16.

The data in Tables 8 and 9 indicates that as a feeder

Table 8. Corn and soilage quantities[a] and the marginal rate of substitution for selected stilbestrol rations (temperature is held constant at

Ration (ratio of soilage to corn)	100 lbs. gain			200 lbs. gain			300
	Lbs.[b] soilage	Lbs.[c] corn	$\frac{\partial F}{\partial C}$[d]	Lbs. soilage	Lbs. corn	$\frac{\partial F}{\partial C}$	Lbs. soilage
All soilage	5,182	0	7.55				
20:1	3,834	192	6.57	9,028	451	11.55	
15:1	3,545	236	6.38	8,087	539	10.01	
10:1	3,091	309	6.11	6,799	680	8.42	11,777[e]
8:1	2,825	353	5.96	6,117	765	7.74	10,228[e]
5:1	2,256	451	5.65	4,766	953	6.65	7,639
3:1	1,672	557	5.36	3,480	1,160	5.82	5,463
2:1	1,268	634	5.17	2,626	1,313	5.36	4,095

[a]For each of the feed combinations there would also be fed a certain am be fed at the rate of .2 of a pound per day. The estimated number of feeding in Table 24.

[b]The all soilage value was derived from equation 21, all other values we

[c]Derived from equation 27.

[d]The marginal rate of substitution of corn for soilage.

[e]The estimated feeding period exceeds the 130 day average feeding period

ution along the 100, 200, 300, 350 and 400 pound beef gain isoquants
nt at the overall mean)

300 lbs. gain			350 lbs. gain			400 lbs. gain		
. age	Lbs. corn	$\frac{\partial F}{\partial C}$	Lbs. soilage	Lbs. corn	$\frac{\partial F}{\partial C}$	Lbs. soilage	Lbs. corn	$\frac{\partial F}{\partial C}$
77[e]	1,178	16.61						
28[e]	1,278	13.07	12,891[e]	1,611	25.19			
39	1,528	8.57	9,275	1,855	10.44	11,105[e]	2,221	14.15
63	1,821	6.52	6,539	2,180	7.03	7,685[e]	2,562	7.72
95	2,046	5.61	4,881	2,441	5.77	5,709	2,854	5.97

in amount of the supplement shown in Table 3. This supplement would
eeding days for each of the feed combinations in this table are shown

ues were derived using equation 26.

period in the experiment.

Table 9. Corn and soilage quantities[a] and marginal rates of substitution gain isoquants for selected non-stilbestrol rations (temperatur

Ration (ratio of soilage to corn)	100 lbs. gain			200 lbs. gain			3
	Lbs.[b] soilage	Lbs.[c] corn	$\frac{\partial F}{\partial C}$[d]	Lbs. soilage	Lbs. corn	$\frac{\partial F}{\partial C}$	Lbs. soilag
All soilage	5,526	0	8.64				
20:1	3,896	195	8.11	9,401	470	11.86	
15:1	3,556	237	8.01	8,386	559	10.99	
10:1	3,031	303	7.86	6,956	696	10.01	12,445
8:1	2,732	341	7.78	6,190	774	9.57	11,898
5:1	2,111	422	7.62	4,687	937	8.84	8,328
3:1	1,507	502	7.46	3,299	1,100	8.26	5,650
2:1	1,111	556	7.36	2,417	1,208	7.97	4,078

[a]For each of the feed combinations there would also be fed a certain Table 3. This supplement would be fed at the rate of .2 of a pound per d days for each of the feed combinations in this table are shown in Table 2

[b]The all soilage value was derived from equation 22, all other value

[c]Derived from equation 28.

[d]The marginal rate of substitution of corn for soilage.

[e]The estimated feeding period exceeds the 130 day average feeding pe

s of substitution along the 100, 200, 300 and 350 pound beef
tions (temperature is held constant at the overall mean)

...ain	300 lbs. gain			350 lbs. gain		
$\frac{\partial F}{\partial C}$	Lbs. soilage	Lbs. corn	$\frac{\partial F}{\partial C}$	Lbs. soilage	Lbs. corn	$\frac{\partial F}{\partial C}$
11.86						
10.79						
10.01	12,445[e]	1,474	50.06			
9.57	11,898[e]	1,487	35.72			
8.84	8,328	1,666	14.43	9,646[e]	2,354	45.25
8.26	5,650	1,883	10.77	7,341[e]	2,447	17.52
7.97	4,078	2,039	9.52	5,195	2,597	12.19

be fed a certain amount of the supplement shown in
of a pound per day. The estimated number of feeding
shown in Table 25.

all other values were derived from equation 26.

ge.

erage feeding period in the experiment.

steer takes on more weight, fed any given ration, the rate of substitution of corn for soilage increases. For example, in Table 8, with the 15:1 ration a feeder steer fed 3,545 pounds of soilage and 236 pounds of corn is predicted to gain 100 pounds and the marginal rate of substitution of corn for soilage will be 6.38. That is, one additional pound of corn replaces only 6.38 pounds of soilage. If the steer is fed the same ration until he consumes 8,087 pounds of soilage and 539 pounds of corn, then it is estimated the steer will have gained 200 pounds and the predicted marginal rate of substitution of corn for soilage will be 10.01.

The increase in the rate of substitution of corn for soilage, along any one ration line, indicates that corn becomes more important in the fattening ration relative to soilage as the feeder steer increases in weight. These results are consistent with the logic presented in a previous section. In the fattening process the formation of fat is dependent upon the amount of total digestible nutrients in the feed. Therefore, as feeder animals increase in weight, high energy feeds, such as corn, become more valuable relative to forages in the fattening process.

Ration Lines

The production surface may be further examined by investigating the input-output relationships when the two feeds -- corn and soilage -- are fed in fixed proportions. Since, for any given ration line, the two feeds -- corn and soilage -- are held in fixed proportions, it is possible to derive feed-gain transformation equations from the production functions. The feed-gain transformation equations are derived by defining a new variable, γ, as the total pounds of feed of a given ration. Then, for each fixed ration, each feed input variable is redefined in terms of γ and substituted into the production function equation to give the feed-gain transformation equation or a total gain equation for that particular fixed ration. Thus, the total gain equation for each ration predicts the total amount of gain from various quantities of feed of a fixed ration. The marginal or additional gain equations may be derived from the total gain equations by taking the first derivative of gain (G) with respect to total feed (γ).* The marginal gain equation is used to estimate the additional gain from the last pound of feed fed of a given ration.

Total and marginal gain equations, for eight selected rations, are derived from the overall stilbestrol production function 20 and are shown in Table 10. Similar equations derived from the overall non-stilbestrol production function

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*The method used for deriving the total and marginal gain equations in Tables 10 and 11 is the same method used by Heady <u>et al</u>. (23, pp. 472-474).

Table 10. Total and marginal gain equations, derived from the overall stilbestrol quadratic function, for selected rations for 850 pound good-to-choice feeder steers

Ration[a]	Prediction equations for: Total gain	Marginal gain
Ration A All soilage	$G_A = .023160514\gamma_A{}^{b} - .000000745\gamma_A^2 - 1.223605\ H$	$\frac{\partial G_A}{\partial \gamma_A} = .023160514 - .000001508\gamma_A$
Ration B 20:1	$G_B = .02759913\gamma_B - .000000686\gamma_B^2 - 1.223605\ H$	$\frac{\partial G_B}{\partial \gamma_B} = .02759913 - .000001372\gamma_B$
Ration C 15:1	$G_C = .02898620\gamma_C - .000000672\gamma_C^2 - 1.223605\ H$	$\frac{\partial G_C}{\partial \gamma_C} = .02898620 - .000001344\gamma_C$
Ration D 10:1	$G_D = .0316342399\gamma_D - .000000654\gamma_D^2 - 1.223605\ H$	$\frac{\partial G_D}{\partial \gamma_D} = .0316342399 - .000001308\gamma_D$
Ration E 8:1	$G_E = .0335172901\gamma_E - .000000647\gamma_E^2 - 1.223605\ H$	$\frac{\partial G_E}{\partial \gamma_E} = .0335172901 - .000001294\gamma_E$
Ration F 5:1	$G_F = .0386956787\gamma_F - .000000651\gamma_F^2 - 1.223605\ H$	$\frac{\partial G_F}{\partial \gamma_F} = .0386956787 - .000001302\gamma_F$
Ration G 3:1	$G_G = .0464632605\gamma_G - .000000724\gamma_G^2 - 1.223605\ H$	$\frac{\partial G_G}{\partial \gamma_G} = .0464632605 - .000001448\gamma_G$
Ration H 2:1	$G_H = .0542308423\gamma_H - .000000878\gamma_H^2 - 1.223605\ H$	$\frac{\partial G_H}{\partial \gamma_H} = .0542308423 - .000001756\gamma_H$

[a]Ration is the ratio of soilage to corn.

[b]γ denotes total pounds of feed of the particular ration.

21 are shown in Table 11.

The predicted total gain values for various levels of feed input, for the eight selected rations, are shown in Table 12 and plotted in Figure 17 for beef steers fed stilbestrol. The estimated marginal gain values corresponding to the total gain values are presented in Table 13. Similarly, the predicted total gain values for non-stilbestrol fed steers are shown in Table 14 and plotted in Figure 18 and the associated marginal gains are shown in Table 15. The predicted total gain values in both Tables 12 and 13 show that from the same total pounds of feed total gain is monotonically increasing as the proportion of corn in the ration increases. In Table 12, 7,000 pounds of feed of an all soilage ration is predicted to produce 125.6 pounds of gain, whereas, if the ration is the 20:1 ration then the 7,000 pounds of feed will produce 159.6 pounds of beef gains. The other columns in Table 12 are interpreted in the same manner.

The marginal gain indicates the amount of gain added to total gain from the last pound of feed fed. For any given ration, the marginal gains, as shown in Tables 13 and 15, are monotonically decreasing, indicating diminishing returns to feed. For the 10:1 ration, in Table 13, the 2,000th pound of feed adds .0290 pounds of gain to total gain while the 10,000th pound of feed adds only .0185 pounds of gain. For any one level of feed fed the marginal productivity of feed

Table 11. Total and marginal gain equations, derived from the overall non-stilbestrol quadratic function, for selected rations for 850 pound good-to-choice feeder steers

Ration[a]	Prediction equations for: Total gain	Marginal gain
Ration A All soilage	$G_A = .021287744\,\gamma_A{}^{b} - .000000578\,\gamma_A^2 - 2.2005042\,H$	$\frac{\partial G_A}{\partial \gamma_A} = .021287744 - .000001156\,\gamma_A$
Ration B 20:1	$G_B = .0274034765\,\gamma_B - .000000724\,\gamma_B^2 - 2.2005042\,H$	$\frac{\partial G_B}{\partial \gamma_B} = .0274034765 - .000001448\,\gamma_B$
Ration C 15:1	$G_C = .0293146425\,\gamma_C - .000000777\,\gamma_C^2 - 2.2005042\,H$	$\frac{\partial G_C}{\partial \gamma_C} = .0293146425 - .000001556\,\gamma_C$
Ration D 10:1	$G_D = .0329632326\,\gamma_D - .000000892\,\gamma_D^2 - 2.2005042\,H$	$\frac{\partial G_D}{\partial \gamma_D} = .0329632326 - .000001784\,\gamma_D$
Ration E 8:1	$G_E = .0355577856\,\gamma_E - .000000982\,\gamma_E^2 - 2.2005042\,H$	$\frac{\partial G_E}{\partial \gamma_E} = .0355577856 - .000001964\,\gamma_E$
Ration F 5:1	$G_F = .0426928071\,\gamma_F - .000001268\,\gamma_F^2 - 2.2005042\,H$	$\frac{\partial G_F}{\partial \gamma_F} = .0426928071 - .000002536\,\gamma_F$
Ration G 3:1	$G_G = .0533953380\,\gamma_G - .000001802\,\gamma_G^2 - 2.2005042\,H$	$\frac{\partial G_G}{\partial \gamma_G} = .0533953380 - .000003604\,\gamma_G$
Ration H 2:1	$G_H = .0640978689\,\gamma_H - .000002461\,\gamma_H^2 - 2.2005042\,H$	$\frac{\partial G_H}{\partial \gamma_H} = .0640978689 - .000004922\,\gamma_H$

[a]Ration is the ratio of soilage to corn.

[b]γ denotes total pounds of feed of the particular ration.

Table 12. Estimated total gain from various total feed quantities[a] (γ) of selected stilbestrol rations fed to 850 pound good-to-choice feeder steers[b]

Pounds of feed fed	Total gain[c] in pounds for selected rations:[d]							
	All soilage	20:1	15:1	10:1	8:1	5:1	3:1	2:1
500	11.4	13.6	14.3	15.7	16.6	19.2	23.1	26.9
1,000	22.4	26.9	28.3	31.0	32.9	38.0	45.7	53.4
1,500	33.1	39.9	42.0	46.0	48.8	56.6	68.1	79.4 e
2,000	43.3	52.5	55.3	60.7	64.4	74.8	90.0	104.9
2,500	53.2	64.7	68.3	75.0	79.7	92.7	111.6	130.1
3,000	62.8	76.6	80.9	89.0	94.7	110.2	132.9	154.8
3,500	71.9	88.2	93.2	102.7	109.4	127.5	153.7	179.1 f
4,000	80.7	99.4	105.2	116.1	123.7	144.4	174.3	202.9
4,500	89.1	110.3	116.8	129.1	137.7	160.9	194.4	226.3
5,000	97.2	120.9	128.1	141.8	151.4	177.2	214.2	249.2
5,500	104.8	131.1	139.1	154.2	164.8	193.1	233.6	271.7 g
6,000	112.1	140.9	149.7	166.3	177.8	208.7	252.7	293.8
6,500	119.0	150.4	160.0	178.0	190.5	224.0	271.4	315.4
7,000	125.6	159.6	170.0	189.4	202.9	239.0	289.7	336.6
7,500	131.8	168.4	179.6	200.5	215.0	253.6	307.7	357.3 h
8,000	137.6	176.9	188.8	211.2	226.7	267.9	325.3	377.7
8,500	143.0	185.0	197.8	221.6	238.2	281.9	342.6	397.5 i
9,000	148.1	192.8	206.4	231.7	249.3	295.5	359.5	417.0
9,500	152.7	200.3	214.7	241.5	260.0	308.8	376.0	435.9
10,000	157.1	207.4	222.6	250.9	270.5	321.8	392.2	454.5
10,500	161.0	214.2	230.2	260.0	280.6	334.5	408.0	472.6
11,000	164.6	220.6	237.5	268.8	290.4	346.9	423.4	
11,500	167.8	226.7	244.4	277.3	299.9	358.9		
12,000	170.6	232.4	251.0	285.4	309.0	370.6		
12,500	173.0	237.8	257.2	293.2	317.9			

11,500	167.8	226.7	244.4	277.3	299.9	358.9	
12,000	170.6	232.4	251.0	285.4	309.0	370.6	
12,500	173.0	237.8	257.2	293.2	317.9		
13,000	175.1	242.9	263.2	300.7			
13,500	176.8	247.6	268.7				
14,000	178.1						
14,500	179.1						
15,000	179.7						

[a]In addition to the feed fed of selected rations there would also be fed a certain amount of the supplement shown in Table 3. This supplement would be fed at the rate of .2 of a pound per day. The estimated number of feeding days for each of the feed quantities is shown in Table 30.

[b]Temperature is held constant at the overall mean.

[c]All values are derived from the equations in Table 10.

[d]The ration is the ratio of soilage to corn.

[e]The horizontal line indicates a 30 day feeding period (see Table 29).

[f]The horizontal line indicates a 60 day feeding period (see Table 29).

[g]The horizontal line indicates a 90 day feeding period (see Table 29).

[h]The horizontal line indicates a 120 day feeding period (see Table 29).

[i]The horizontal line indicates a 140 day feeding period (see Table 29).

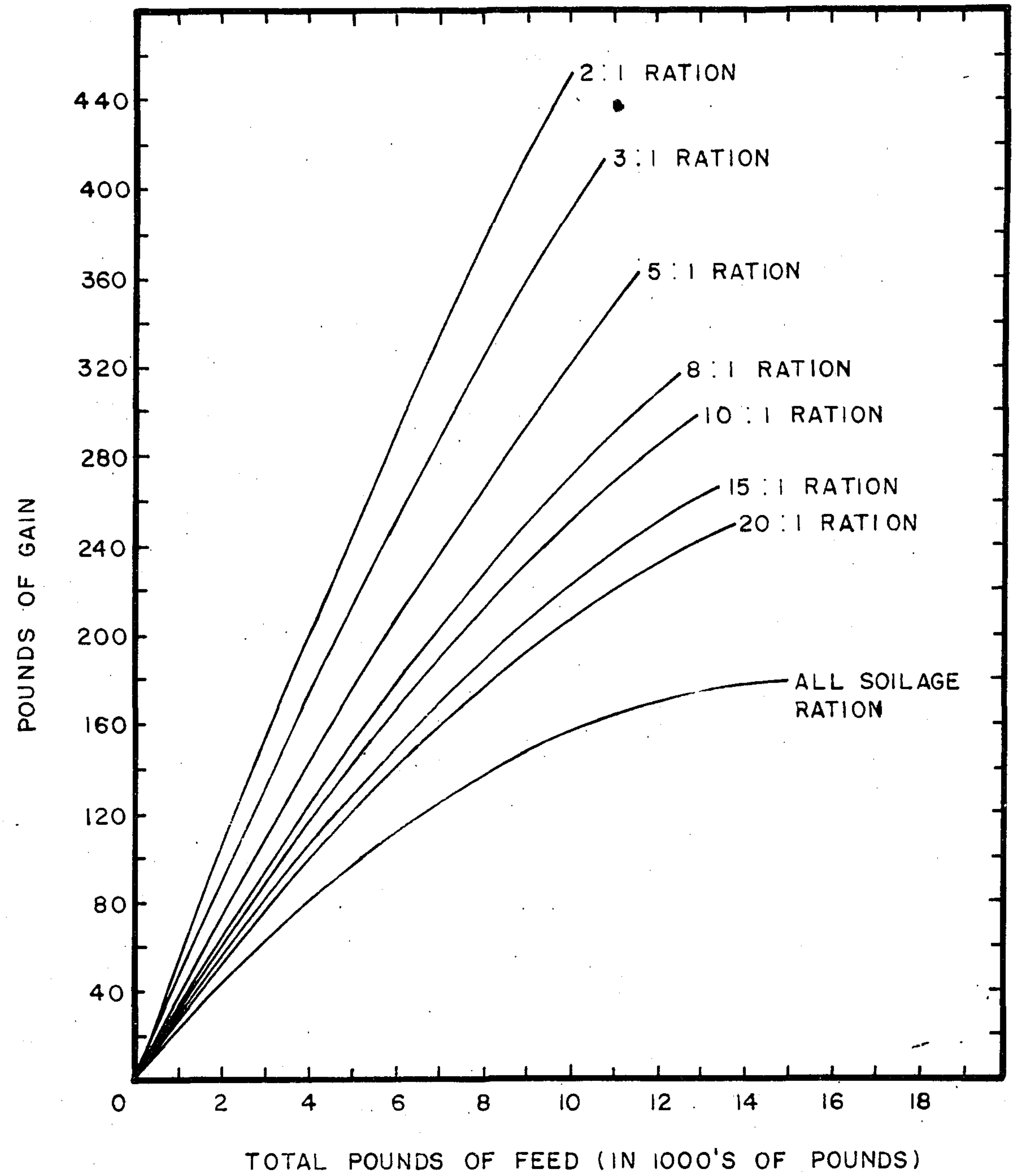

Figure 17. Feed-gain transformation curves for selected rations derived from the overall stilbestrol production function (temperature held constant at the overall mean)

Table 13. Estimated marginal gain from various total feed quantities (γ) of selected soilage-corn ration fed to 850 pound good-to-choice feeder steers (with stilbestrol)

Pounds of feed fed	Marginal gain[a] in pounds for selected rations:[b]							
	All soilage	20:1	15:1	10:1	8:1	5:1	3:1	2:1
500	.0224	.0269	.0283	.0310	.0329	.0380	.0457	.0534
1,000	.0217	.0262	.0276	.0303	.0322	.0374	.0450	.0525
1,500	.0209	.0255	.0270	.0297	.0316	.0367	.0442	.0516 c
2,000	.0202	.0249	.0263	.0290	.0309	.0361	.0436	.0507
2,500	.0194	.0242	.0256	.0284	.0303	.0354	.0428	.0498
3,000	.0187	.0235	.0250	.0277	.0296	.0348	.0421	.0490
3,500	.0179	.0228	.0243	.0271	.0290	.0341	.0414	.0481 d
4,000	.0171	.0221	.0236	.0264	.0283	.0335	.0407	.0472
4,500	.0165	.0214	.0229	.0257	.0277	.0328	.0399	.0463
5,000	.0157	.0207	.0223	.0251	.0270	.0322	.0392	.0455
5,500	.0150	.0201	.0216	.0244	.0264	.0315	.0385	.0446
6,000	.0142	.0194	.0209	.0238	.0258	.0309	.0378	.0437 e
6,500	.0135	.0187	.0202	.0231	.0251	.0302	.0370	.0428
7,000	.0127	.0180	.0196	.0225	.0245	.0296	.0363	.0419
7,500	.0120	.0173	.0189	.0218	.0238	.0289	.0356	.0411
8,000	.0112	.0166	.0182	.0212	.0232	.0283	.0349	.0402
8,500	.0105	.0159	.0176	.0205	.0225	.0276	.0341	.0393 f
9,000	.0097	.0153	.0169	.0199	.0219	.0270	.0334	.0384
9,500	.0090	.0146	.0162	.0192	.0212	.0263	.0327	.0375
10,000	.0083	.0139	.0155	.0185	.0206	.0257	.0320	.0367
10,500	.0075	.0132	.0149	.0179	.0199	.0250	.0312	.0358
11,000	.0068	.0125	.0142	.0172	.0193	.0244	.0305	
11,500	.0060	.0118	.0135	.0166	.0186	.0237		
12,000	.0053	.0111	.0128	.0159	.0180	.0230		
12,500	.0045	.0105	.0122	.0153	.0173			
13,000	.0038	.0098	.0115	.0146				
13,500	.0030	.0091	.0108					
14,000	.0023							
14,500	.0015							
15,000	.0008							

8,500	.0105	.0159	.0176	.0205	.0225	.0276	.0341	.0393 [f]
9,000	.0097	.0153	.0169	.0199	.0219	.0270	.0334	.0384
9,500	.0090	.0146	.0162	.0192	.0212	.0263	.0327	.0375
10,000	.0083	.0139	.0155	.0185	.0206	.0257	.0320	.0367
10,500	.0075	.0132	.0149	.0179	.0199	.0250	.0312	.0358
11,000	.0068	.0125	.0142	.0172	.0193	.0244	.0305	
11,500	.0060	.0118	.0135	.0166	.0186	.0237		
12,000	.0053	.0111	.0128	.0159	.0180	.0230		
12,500	.0045	.0105	.0122	.0153	.0173			
13,000	.0038	.0098	.0115	.0146				
13,500	.0030	.0091	.0108					
14,000	.0023							
14,500	.0015							
15,000	.0008							

[a]All values are derived from the equations in Table 10.

[b]The ration is the ratio of soilage to corn.

[c]The horizontal line indicates 100 pounds of gain (see Table 11).

[d]The horizontal line indicates 200 pounds of gain (see Table 11).

[e]The horizontal line indicates 300 pounds of gain (see Table 11).

[f]The horizontal line indicates 400 pounds of gain (see Table 11).

Table 14. Estimated total gain from various total feed quantities[a] (γ) of selected soilage-corn rations fed to 850 pound good-to-choice feeder steers (without stilbestrol)[b]

Pounds of feed fed	Total gain[c] in pounds for selected rations:[d]							
	All soilage	20:1	15:1	10:1	8:1	5:1	3:1	2:1
500	10.5	13.5	14.5	16.3	17.5	21.0	26.2	31.4
1,000	20.7	26.7	28.5	32.1	34.6	41.4	51.6	61.6
1,500	30.6	39.5	42.2	47.4	51.1	61.2	76.0	90.6 e
2,000	40.3	51.9	55.5	62.4	67.2	80.3	99.6	118.4
2,500	49.6	64.0	68.4	76.8	82.8	98.8	122.2	144.9
3,000	58.7	75.7	80.9	90.9	97.8	116.7	144.0	170.1
3,500	67.4	87.0	93.1	104.4	112.4	133.9	164.8	194.2 f
4,000	75.9	98.0	104.8	117.6	126.5	150.5	184.8	217.0
4,500	84.1	108.7	116.2	130.3	140.1	166.4	203.8	238.6
5,000	92.0	118.9	127.1	142.5	153.2	181.8	221.9	259.0 g
5,500	99.6	128.8	137.7	154.3	165.9	196.4	239.1	278.1
6,000	106.9	138.4	147.9	165.7	178.0	210.5	255.5	296.0
6,500	114.0	147.6	157.7	176.6	189.6	223.9	270.9	312.6
7,000	120.7	156.4	167.1	187.0	200.8	236.7	285.5	328.1 h
7,500	127.2	164.8	176.1	197.1	211.4	248.9	299.1	342.3
8,000	133.3	172.9	184.7	206.6	221.6	260.4	311.8	355.3
8,500	139.2	180.7	193.0	215.7	231.3	271.3	323.7	367.0 i
9,000	144.8	188.0	200.8	224.4	240.5	281.5	334.6	377.5
9,500	150.1	195.0	208.3	232.7	249.2	291.1	344.6	386.8
10,000	155.1	201.7	215.4	240.4	257.4	300.1	353.8	394.8
10,500	159.8	208.0	222.1	247.8	265.1	308.5	362.0	401.7
11,000	164.3	213.9	228.4	254.7	272.3	316.2	369.3	
11,500	168.4	219.4	234.3	261.1	279.0	323.3	375.7	
12,000	172.3	224.6	239.8	267.1	285.3	329.7		
12,500	175.9	229.5	244.9	272.7	291.0	335.5		
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]			

12,500	175.9	229.5	244.9	272.7	291.0	335.5
13,000	179.1	234.0	249.7	277.8	296.3	
13,500	182.1	238.1	254.0	282.5	301.0	
14,000	184.8	241.8	258.0	286.7	305.3	
14,500	187.2	245.2	261.6			
15,000	189.4	248.2	264.8			
15,500	191.2	250.9				
16,000	192.8					
16,500	194.0					
17,000	195.0					

[a]In addition to the feed fed there would also be fed a certain amount of the supplement shown in Table 3. This supplement would be fed at the rate of .2 of a pound per day. The estimated number of feeding days for each of the feed quantities is shown in Table 31.

[b]Temperature is held constant at the overall mean.

[c]All values are derived from the equations in Table 11.

[d]The ration is the ratio of soilage to corn.

[e]The horizontal line indicates a 30 day feeding period (see Table 30).

[f]The horizontal line indicates a 60 day feeding period (see Table 30).

[g]The horizontal line indicates a 90 day feeding period (see Table 30).

[h]The horizontal line indicates a 120 day feeding period (see Table 30).

[i]The horizontal line indicates a 140 day feeding period (see Table 30).

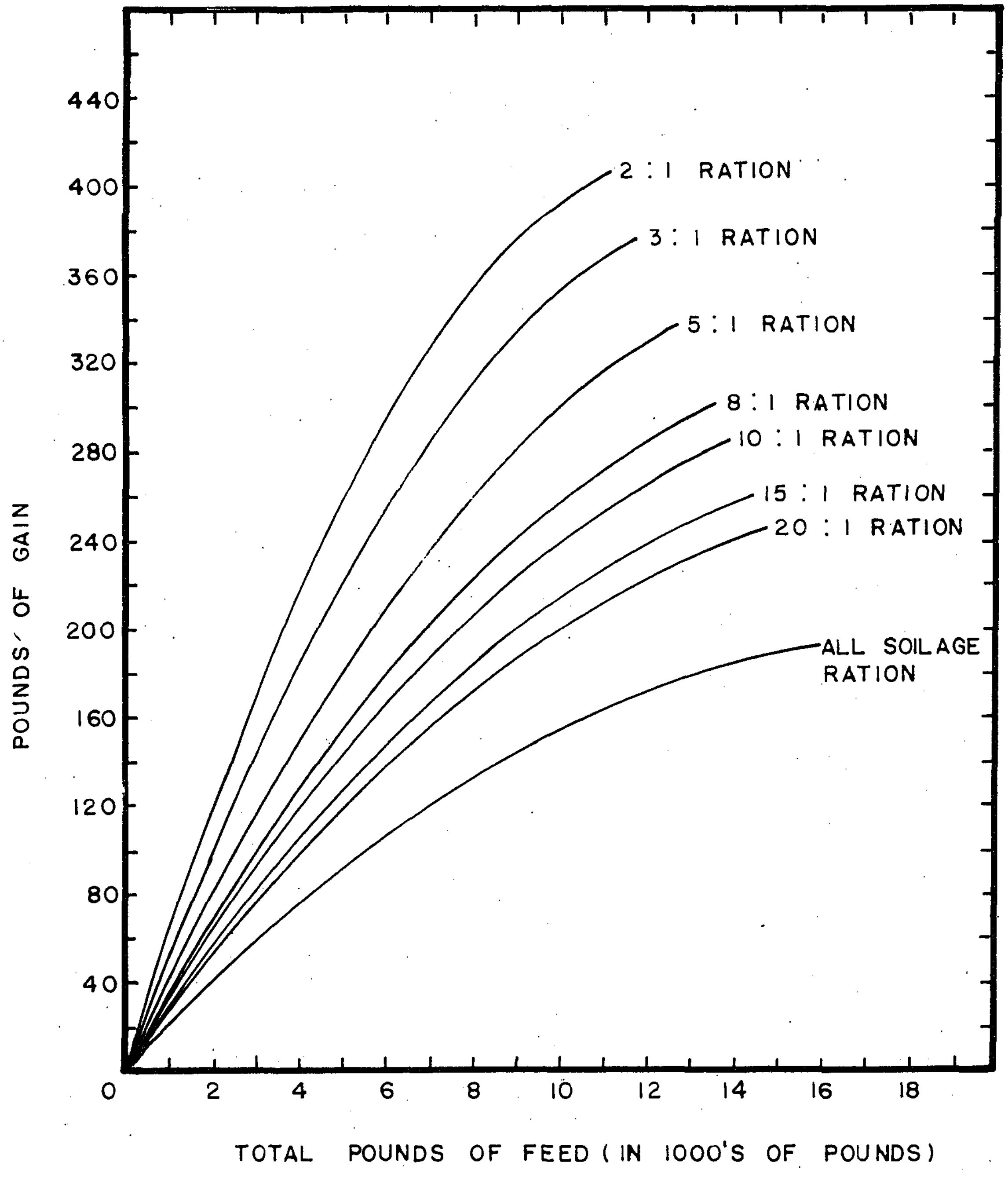

Figure 18. Feed-gain transformation curves for selected rations derived from the overall non-stilbestrol production function (temperature held constant at the overall mean)

Table 15. Estimated marginal gain from various total feed quantities (γ) of selected soilage-corn rations fed to 850 pound good-to-choice feeder steers (without stilbestrol)

Pounds of feed fed	Marginal gain[a] in pounds for selected rations:[b] All soilage	20:1	15:1	10:1	8:1	5:1	3:1	2:1
500	.0207	.0267	.0285	.0321	.0346	.0414	.0516	.0616
1,000	.0201	.0260	.0278	.0312	.0336	.0402	.0498	.0592
1,500	.0200	.0252	.0270	.0303	.0326	.0389	.0480	.0567 c
2,000	.0190	.0245	.0262	.0294	.0316	.0376	.0462	.0543
2,500	.0184	.0238	.0254	.0285	.0306	.0364	.0444	.0518
3,000	.0178	.0231	.0246	.0276	.0297	.0351	.0426	.0493
3,500	.0172	.0223	.0239	.0267	.0287	.0338	.0408	.0469 d
4,000	.0167	.0216	.0231	.0258	.0277	.0325	.0390	.0444
4,500	.0161	.0209	.0223	.0249	.0267	.0313	.0372	.0419
5,000	.0155	.0202	.0215	.0240	.0257	.0300	.0354	.0395
5,500	.0149	.0194	.0208	.0232	.0248	.0287	.0336	.0370
6,000	.0144	.0187	.0200	.0223	.0238	.0275	.0318	.0346 e
6,500	.0138	.0180	.0192	.0214	.0228	.0262	.0300	.0321
7,000	.0132	.0173	.0184	.0205	.0218	.0249	.0282	.0296
7,500	.0126	.0165	.0177	.0196	.0208	.0237	.0264	.0272 f
8,000	.0120	.0158	.0169	.0187	.0198	.0224	.0246	.0247
8,500	.0115	.0151	.0161	.0178	.0189	.0211	.0228	.0223
9,000	.0109	.0144	.0153	.0169	.0179	.0199	.0210	.0198
9,500	.0103	.0137	.0145	.0160	.0169	.0186	.0192	.0173
10,000	.0097	.0129	.0138	.0151	.0159	.0173	.0174	.0149 g
10,500	.0092	.0122	.0130	.0142	.0149	.0161	.0156	.0124
11,000	.0086	.0115	.0122	.0133	.0140	.0148	.0138	
11,500	.0080	.0108	.0114	.0124	.0130	.0135	.0120	
12,000	.0074	.0100	.0107	.0116	.0120	[illegible]		

[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.0124
11,000	.0086	.0115	.0122	.0133	.0140	.0148	.0138	
11,500	.0080	.0108	.0114	.0124	.0130	.0135	.0120	
12,000	.0074	.0100	.0107	.0116	.0120	.0122		
12,500	.0068	.0093	.0099	.0107	.0110	.0110		
13,000	.0063	.0086	.0091	.0097	.0100			
13,500	.0057	.0079	.0083	.0089	.0090			
14,000	.0051	.0071	.0075	.0080	.0081			
14,500	.0045	.0064	.0068					
15,000	.0040	.0057	.0060					
15,500	.0034	.0050						
16,000	.0028							
16,500	.0022							
17,000	.0017							

[a]All values have been derived from the equations in Table 11.

[b]The ration is the ratio of soilage to corn.

[c]The horizontal line indicates 100 pounds of gain (see Table 13).

[d]The horizontal line indicates 200 pounds of gain (see Table 13).

[e]The horizontal line indicates 300 pounds of gain (see Table 13).

[f]The horizontal line indicates 350 pounds of gain (see Table 13).

[g]The horizontal line indicates 400 pounds of gain (see Table 13).

increases as the proportion of corn in the ration increases.

Least-Cost Rations

The least-cost ration or combination of corn and soilage for producing a given level of gain is specified whenever the marginal rate of substitution between the feeds is equal to their inverse price ratio. That is, the least-cost ration for a given level of gain is determined when

(29) $\frac{\partial F}{\partial C} = \frac{P_C}{P_F}$, where $\frac{\partial F}{\partial C}$ is the marginal rate of substitution of corn for soilage and P_C is the price of corn and P_F is the price of soilage. Whenever the substitution ratio is greater than the price ratio, a ration containing a greater proportion of corn should be fed because the value of the soilage replaced by corn is greater than the value of the added corn. However, if the substitution ratio is less than the price ratio then a ration containing less corn should be fed in order to attain the given level of gain with the least-cost ration.

The beef steers in this experiment were fed a fixed ration throughout the course of the experiment. Therefore, the production function, which expresses total gains as a function of the corn and soilage consumed, is determined for a feeding system where all rations are a fixed proportion of corn and soilage and are fed as such for the entire feeding period. Furthermore, the isoquants, derived from the production function, show all the possible combinations of corn and soilage that will produce a given level of gain under a fixed ration feeding system.

Since the slope of the gain isoquant is the marginal rate of substitution between feeds, then the point on the gain isoquant where the substitution ratio is equal to the inverse price ratio specifies a certain combination of corn and soilage that will produce the given level of gain at least cost.* This combination of corn and soilage determines the ration, that when fed as a fixed ration, will produce the given level of gain at least cost. The ration is found by solving the particular combination of corn and soilage for the ratio of soilage to corn. However, for any other level of gain, assuming the feed price ratio unchanged, this ration will not be the least-cost ration.

---

*The isocline equations for the stilbestrol and non-stilbestrol rations are (where K is the corn-soilage price ratio):

I. With stilbestrol

$$F = \frac{.02316051K - .11637150 + (.0000000374K + .0000099910)C}{.0000000374 + .000001491K}$$

II. Without stilbestrol

$$F = \frac{.02128774K - .14971812 + (-.0000037907K + .0000245224)C}{-.0000037907 + .000001155K}$$

Various corn prices ranging from 75 cents per bushel through $1.75 per bushel and soilage prices ranging from $1.00 per ton through $8.00 per ton were used in estimating least-cost rations in terms of beef gains. The soilage prices are derived values estimated under different economic circumstances. A soilage price of $6.32 per ton has been estimated (36) to be the cost per ton for soilage delivered to the feed bunk. This estimated price includes an estimated cost of $3.02 per ton for harvesting the soilage plus a value for the green forage standing in the field. The value assigned to the green forage standing in the field is based on $20.00 per ton of harvested hay. If the price of harvested hay is varied, then it is possible to obtain a series of soilage prices such that each soilage price is based upon a different harvested hay price. Furthermore, if forages are included in the crop rotation because they are considered necessary for good land use, then the forage crops may be grown whether or not they are utilized by livestock. In this case the price of soilage may be considered to be made up of only the harvesting costs, that is, $3.02 per ton. However, since the harvesting costs of $3.02 per ton are based on 50 head of cattle eating 80 pounds of soilage per day for 100 days, the harvesting costs may well vary above or below $3.02 per ton depending upon the size of the feeding operation and the harvesting methods. Hence, under different economic circumstances it is possible for soilage prices to vary over a range somewhat greater than from $3.02 to $6.32. Therefore, soilage prices ranging from $1.00 to $8.00 per ton were used in the analysis of least-cost rations, in order to allow for various soilage prices due to the "opportunity costs" of harvested hay and for differences in the cost of harvesting soilage.

Predicted least-cost rations, for various levels of gain, for a series of corn-soilage price ratios are presented in Table 16 for the stilbestrol rations and the least-cost rations for the non-stilbestrol rations are given in Table 17. The least-cost ration when stilbestrol is fed in the ration can be determined in Table 16 in the following manner. If the price of corn is $1.12 per bushel and the price of soilage is $4.00 per ton, then the corn-soilage price ratio is 10.0. For this corn-soilage price ratio, 100 pounds of gain can be produced at least cost by feeding 5,182 pounds of the all soilage ration. The time required for the beef steer to gain 100 pounds is estimated to be 59.5 days.* Using the same price ratio, 200 pounds of gain can be produced by feeding 539 pounds of corn and 8,084 pounds of soilage which is a soilage-corn ration of 15:1. The estimated time required for the beef steer to attain this gain is 105.7 days.* If

*The time equation, equation 39, presented in a later section provides the basis for the time estimates.

Table 16. Least-cost rations for selected levels of gain and various corn-soi (with stilbestrol)[a]

Corn-soilage price ratio	100 lbs. gain: Feed required:[b] Lbs. corn	Lbs. soilage	Ration[c]	No. of days[c]	200 lbs. gain: Feed required: Lbs. corn	Lbs. soilage	Ration	No. of days
5.0	(634)	(1,26[illegible])	(2.0)[e]	(30.5)	(1,313)	(2,626)	(2.0)[e]	(52.[illegible])
6.0	340	2,902	[illegible].5	42.[illegible]	1,110	3,777	3.4	72.0
7.0	99	4,465	45.2	54.2	885	5,234	5.9	83.4
8.0	--	5,182	all soilage	5[illegible].5	730	6,388	8.7	92.4
9.0	--	"	"	"	620	7,31[illegible]	11.[illegible]	[illegible]9.3
10.0	--	"	"	"	539	8,084	15.0	105.7
11.0	--	"	"	"	479	8,722	18.2	110.7
12.0	--	"	"	"	431	9,262	21.5	115.0
13.0	--	"	"	"	394	9,725	24.7	118.7
14.0	--	"	"	"	365	10,125	27.[illegible]	122.0
15.0	--	"	"	"	341	10,474	30.7	12[illegible].[illegible]
16.0	----	"	"	"	321	10,782	33.6	127.4
17.0	--	"	"	"	304	11,094	36.3	129.6
18.0	--	"	"	"	290	11,296	3[illegible].[illegible]	131.7
19.0	--	"	"	"	27[illegible]	11,516	41.4	133.5
20.0	--	"	"	"	268	11,714	43.6	135.2
21.0	--	"	"	"	260	11,893	45.8	136.7[f]
22.0	--	"	"	"	252	12,056	47.8	138.1[f]
23.0	--	"	"	"	245	12,205	49.7	139.4[f]
24.0	--	"	"	"	240	12,342	51.5	140.5[f]
25.0	--	"	"	"	234	12,468	53.2	141.6[f]

[a]Temperature is held constant at the overall mean.

[b]For each of the feed combinations there would also be fed a certain amou fed at the rate of .2 of a pound per day.

[c]Ration is the ratio of soilage to corn.

[d]The time equation, equation 39, presented in a later section provides th

[e]The predicted feed requirements for the least-cost ration resulted in a s of the experiment. Therefore, the highest corn ration (i.e., the 2:1 silage co

[f]The estimated feeding period exceeds the 136 day average feeding period i

corn-soilage price ratios for 850 pound good-to-choice feeder steers

	300 lbs. gain				400 lbs. gain			
No. of days	Feed required: Lbs. corn	Feed required: Lbs. soilage	Ration	No. of days	Feed required: Lbs. corn	Feed required: Lbs. soilage	Ration	No. of days
(62.[illegible])	(2,04[illegible])	(4,095)	(2.0)[e]	(97.5)	(2,354)	(5,709)	(2.0)[e]	(135.2)
72.0	1,940	4,721	2.4	102.9	2,647	5,753	2.0	135.6
[illegible]3.4	1,732	6,064	3.5	114.2	2,659	6,971	2.6	146.7[f]
92.4	1,590	7,127	4.5	123.0	2,530	7,935	3.1	155.4[f]
[illegible]9.6	1,[illegible]	7,[illegible]	5.4	130.1				
105.7	1,414	[illegible],690	6.1	136.0				
110.7	1,35[illegible]	[illegible],279	6.[illegible]	140.9[f]				
115.0	1,315	9,777	7.4	145.1[f]				
118.7	1,260	10,203	[illegible].0	14[illegible].8[f]				
122.0	1,253	10,571	[illegible].4	151.9[f]				
12[illegible].8	1,[illegible]31	10,[illegible]3	[illegible].9	154.7[f]				
127.4								
129.6								
131.7								
133.5								
135.2								
136.7[f]								
138.1[f]								
139.4[f]								
140.5[f]								
141.6[f]								

in amount of the supplement shown in Table 3. This supplement would be

ides the basis for these estimates.

d in a soilage-corn ration of less than 2:1 which is outside the limits
ilage corn ration) has been substituted as the least-cost ration.

period in the experiment.

Table 17. Least-cost rations for selected levels of gain and various corn (without stilbestrol)[a]

Corn-soilage price ratio	100 lbs. gain: Feed required:[b] Lbs. corn	Lbs. soilage	Ration[c]	No. of days[d]	200 lbs. gain: Feed required: Lbs. corn	Lbs. soilage	Ration	No. of days
5.0	(556)	(1,111)	(2.0)[e]	(29.0)	(1,20[illegible])	(2,417)	(2.0)[c]	(61
6.0	"	"	"[e]	"	"	"	"[e]	"
7.0	"	"	"[e]	"	"	"	"[e]	"
8.0	242	3,520	14.6	41.6	1,197	2,502	2.1	62
9.0	--	5,520 all soilage		52.6	898	5,034	5.6	76
10.0	--	"	"	"	69[illegible]	6,936	9.9	87
11.0	--	"	"	"	55[illegible]	8,397	15.0	96
12.0	--	"	"	"	456	9,546	20.[illegible]	103
13.0	--	"	"	"	3[illegible]4	10,[illegible]69	27.3	109
14.0	---	"	"	"	328	11,233	34.2	114
15.0	--	"	"	"	285	11,850	41.6	119
16.0	--	"	"	"	251	12,377	49.[illegible]	122
17.0	--	"	"	"	223	12,827	57.[illegible]	125
18.0	--	"	"	"	201	13,215	65.7	128
19.0	--	"	"	"	183	13,553	74.1	130
20.0	--	"	"	"	168	13,849	82.6	133
21.0	--	"	"	"	155	14,111	91.1	134
22.0	--	"	"	"	144	14,344	99.6	136
23.0	--	"	"	"	135	14,553	10[illegible].0	137
24.0	--	"	"	"	127	14,741	116.3	139
25.0	--	"	"	"	120	14,911	124.5	140

[a]Temperature is held constant at the overall mean.

[b]For each of the feed combinations there would also be fed a certain
fed at the rate of .2 of a pound per day.

[c]Ration is presented as the ratio of soilage to corn.

[d]The time equation, equation 40, presented in a later section provides

[e]The predicted feed requirements for the least-cost ration resulted in
of the experiment. Therefore, the highest corn ration (i.e., the 2:1 soila

[f]The estimated feeding period exceeds the 13[illegible] day average feeding peri

corn-soilage price ratios for 850 pound good-to-choice feeder steers

	300 lbs. gain				350 lbs. gain			
No. of days	Feed required: Lbs. corn	Feed required: Lbs. soilage	Ration	No. of days	Feed required: Lbs. corn	Feed required: Lbs. soilage	Ration	No. of days
(61.7)	(2,039)	(3,247)	(2.0)[e]	(101.5)	(2,597)	(5,195)	(2.0)[e]	(127.1)
"	"	"	"[e]	"	"	"	"[e]	"
"	"	"	"[e]	"	"	"	"[e]	"
62.2	"	"	"[e]	"	"	"	"[e]	"
76.4	"	"	"[e]	"	"	"	"[e]	"
87.8	1,971	4,739	2.4	105.5	"	"	"[e]	"
95.6	1,862	5,885	3.2	112.7	"	"	"[e]	"
103.8	1,7[illegible]3	6,7[illegible]7	3.8	118.6	"	"	"[e]	"
109.[illegible]	1,725	7,510	4.4	123.4	2,559	5,671	2.2	130.3
114.[illegible]	1,6[illegible]1	8,102	4.[illegible]	127.4	2,523	6,162	2.4	133.7
119.0	1,648	8,594	5.2	130.8	2,495	6,569	2.6	136.6
122.6	1,621	9,008	5.6	133.7	2,473	6,913	2.8	139.0[f]
125.8	1,599	9,361	5.9	136.1	2,455	7,205	2.9	141.1[f]
128.5	1,582	9,665	6.1	138.3[f]	2,440	7,456	3.1	142.9[f]
130.9	1,568	9,930	6.3	140.3[f]	2,429	7,677	3.2	144.5[f]
133.0	1,556	10,182	6.5	142.0[f]	2,419	7,870	3.3	145.9[f]
134.9	1,546	10,368	6.7	143.5[f]				
136.7	1,537	10,551	6.9	144.8[f]				
138.2[f]	1,530	10,714	7.0	146.1[f]				
139.6[f]	1,524	10,862	7.1	147.2[f]				
140.8[f]	1,51[illegible]	10,995	7.2	148.2[f]				

in amount of the supplement shown in Table 3. This supplement would be

ides the basis for these estimates.

d in a soilage-corn ration of less than 2:1 which is outside the limits
soilage-corn ration) has been substituted as the least-cost ration.

period in the experiment.

the price of corn is $1.12 per bushel and soilage is $8.00 per ton then the corn-soilage price ratio is 5.0. In Table 16, the predicted least-cost ration for this feed price ratio is less than the 2:1 ration for all levels of gain. Therefore, the highest corn ration fed in the experiment has been substituted for the estimated ration because high corn concentrate rations outisde the limits of the experiment may be physiologically unfeasable. Actually, what the physiologically limiting ration is has not been determined; therefore, in order to not recommend a ration that may be physiologically unfeasable, the 2:1 ration has been used as the least-cost ration in all such cases. Table 17, which gives the least-cost rations for the non-stilbestrol rations, is interpreted in a similar manner.

As mentioned earlier an isocline is a line that connects all points of equal rates of substitution on successive isoquants. The isocline that connects all points on successive isoquants where the substitution ratio is equal to the inverse price ratio specifies at the intersection point with each isoquant the combination of corn and soilage that will produce that level of gain at least cost. Isoclines showing the path of least-cost stilbestrol rations for a few of the corn-soilage price ratios presented in Table 16, are plotted in Figure 19. The least-cost stilbestrol rations for 200, 300 and 400 pounds of gain with a corn-soilage price ratio of 10.0

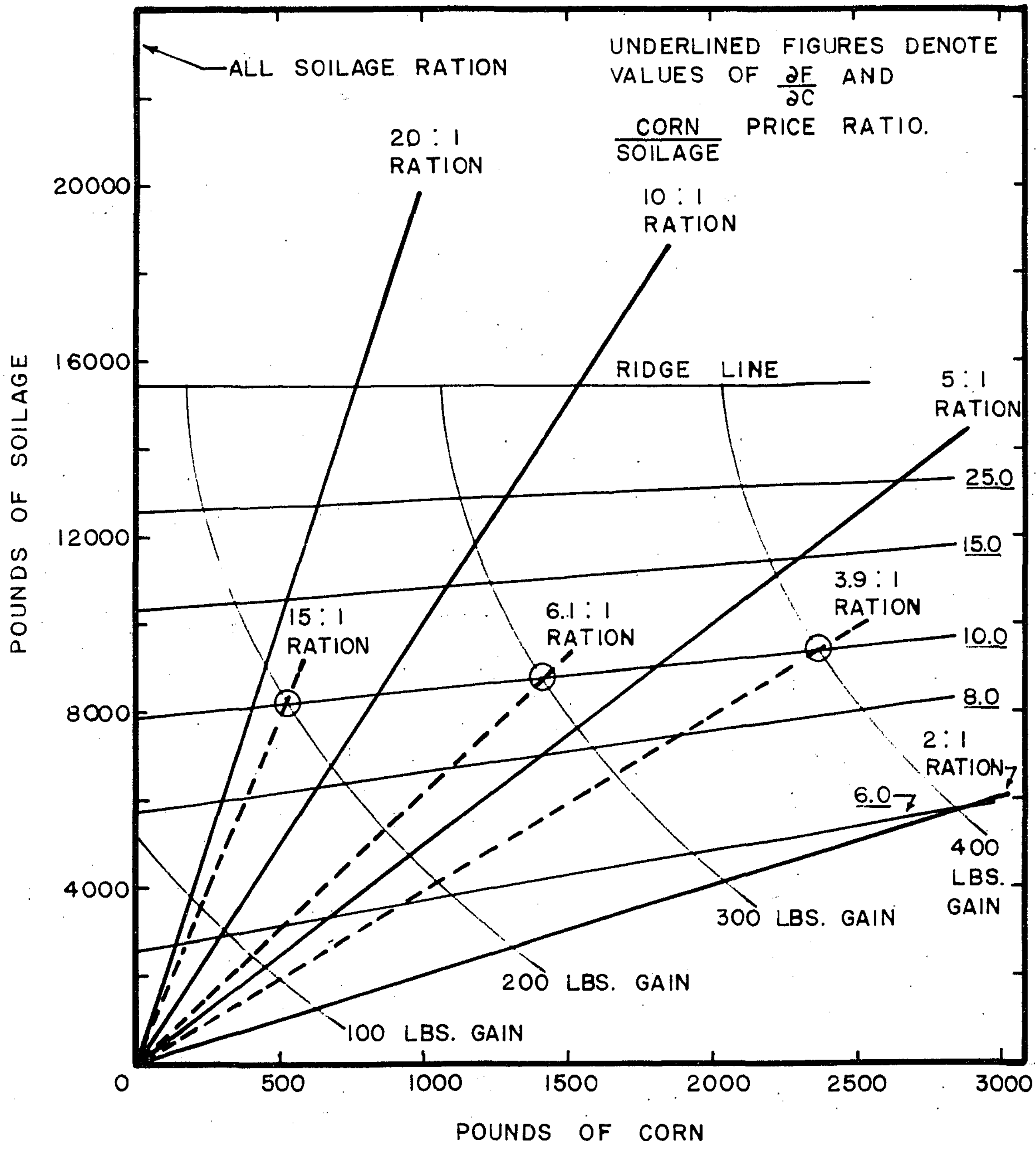

Figure 19. Gain isoquants and isoclines showing the path of least-cost rations for the stilbestrol rations (temperature held constant at the overall mean)

are depicted in Figure 19. For 200 pounds of gain, the least-cost ration is the 15:1 soilage corn ration and for 300 pounds of gain the 6.1:1 soilage-corn ration is the least-cost ration. Isoclines showing the path of least-cost non-stilbestrol rations for some of the corn-soilage price ratios in Table 17 are plotted in Figure 20. Similarly, the least-cost rations for various levels of gain with a corn-soilage price ratio of 10.0 are also shown in Figure 20.

The corn-soilage price ratio map in Figure 21 provides a simplified method of estimating the least-cost ration for various corn and soilage prices. The price ratio map is so designed that it indicates an optimum least-cost ration for a range of corn-soilage price ratios rather than "the" optimum least-cost ration for all possible corn-soilage price ratios. That is, instead of attempting to estimate the least-cost ration for an infinite number of price ratios, only a selected number of least-cost rations have been estimated so that each least-cost ration applies to a range of price ratios. This method of estimating the least-cost rations is based on the assumption that the gain isoquants are made up of a series of linear segments.

The diagonal lines on the price ratio map, in Figure 21, may be called iso-price ratio lines since they depict the various combinations of corn and soilage prices that have the same corn-soilage price ratio. The iso-price ratio lines, in

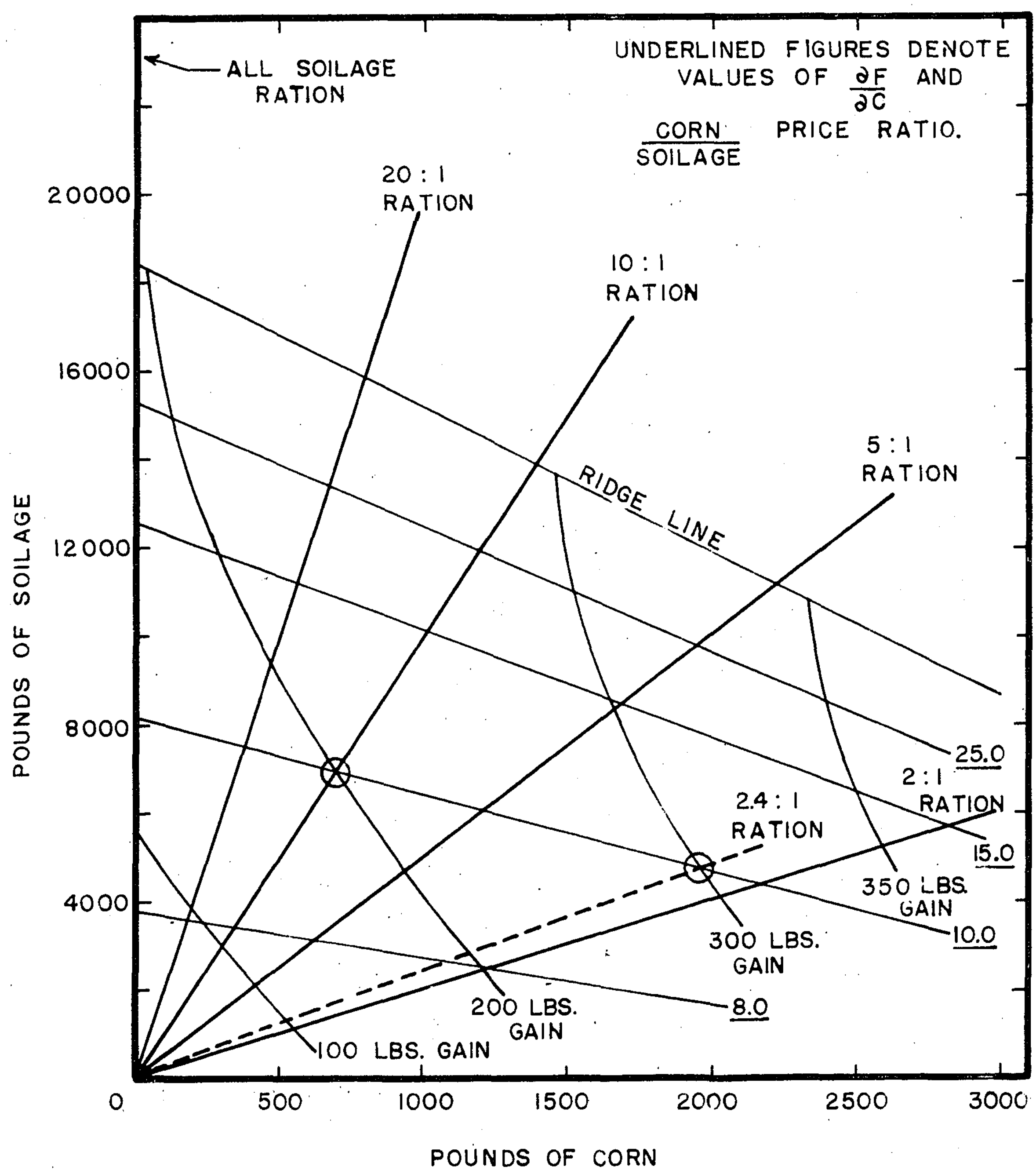

Figure 20. Gain isoquants and isoclines showing the path of least-cost rations for the non-stilbestrol rations (temperature held constant at the overall mean)

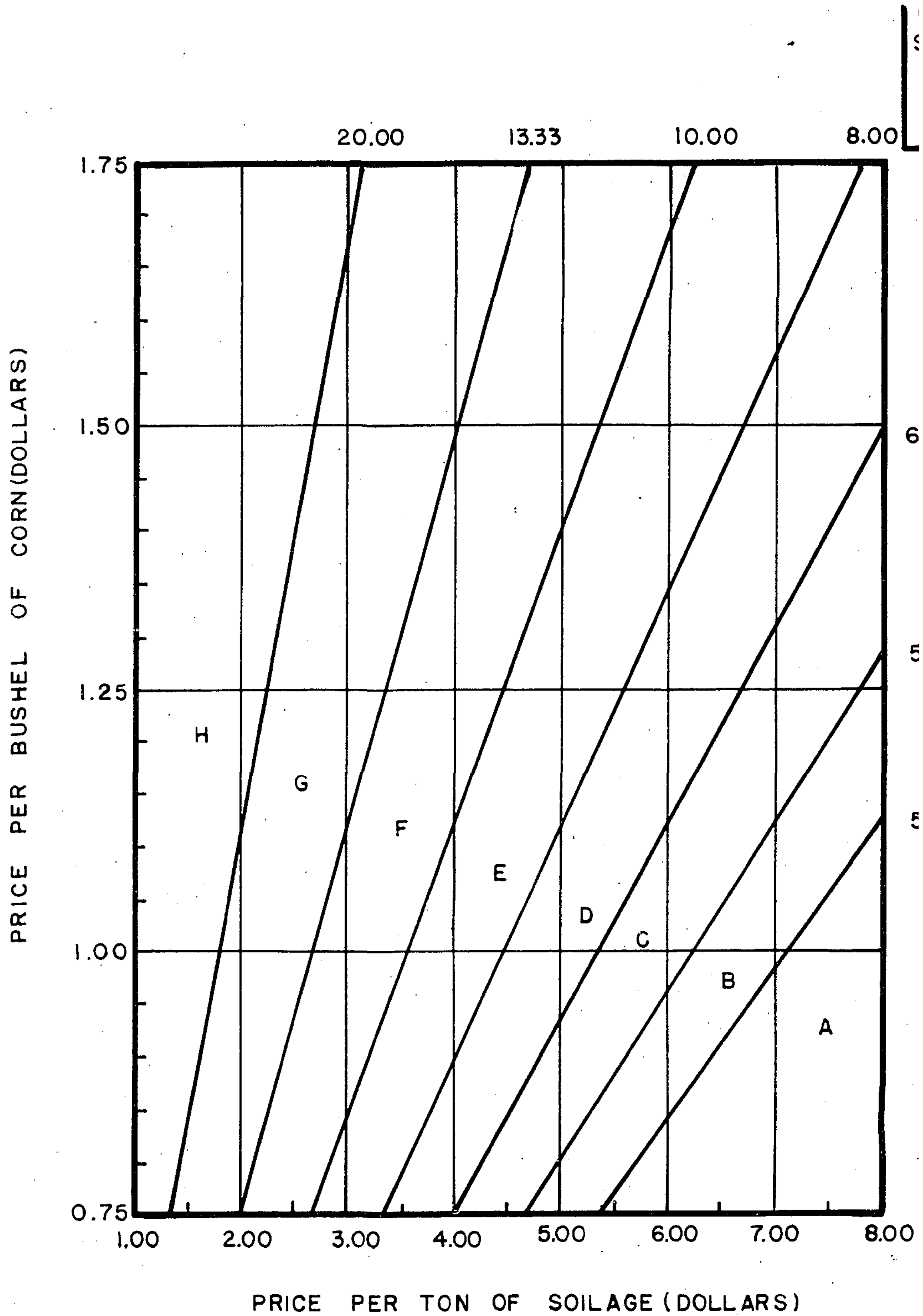

Figure 21. Corn-soilage price ratio map

Figure 21, divide the corn-soilage price map into price ratio areas which are indicated as A, B, ..., H. All price ratios that lie within any one of these price ratio areas specifies the same least-cost ration.

For any given corn price and any given soilage price there exists a unique point on the price ratio map. The price ratio area in which this point lies determines, for several different levels of gain, which of the rations in Table 18 or Table 19 is the least-cost ration.

Given the price of corn and soilage, the level of gain that is to be attained, and whether or not stilbestrol is to be fed, the least-cost ration can be specified in the following manner: Assume the price of corn is $1.20 per bushel and the price of corn is $6.00 per ton. The coordinates of these two prices lies in price ratio area "D". The least-cost ration for 300 pounds of gain when stilbestrol is fed is found in Table 18. The least-cost ration will be a soilage-corn ration of 3.8:1 and requiring 563 pounds of corn and 2,125 pounds of soilage per 100 pounds of gain. This ration requires a feeding period of 117 days* with an average daily gain of 2.57 pounds. The least-cost ration for 100 pounds of gain when stilbestrol is fed in the ration, assuming the same feed price ratio, is found to be a soilage-corn ration of

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*The time equation, equation 39, presented in a later section provides the basis for the time estimate.

Table 18. Least-cost rations for 850 pound good-to-choice feeder steers in terms of feed per 100 pounds of gain (with stilbestrol)[a]

	Price ratio area	Lbs. corn[b]	Lbs. soilage[b]	Ration[c]	Average daily gain[d]	Number of days[e]
100 pounds of gain:	A	(634)	(1,268)	(2.0)[f]	(3.28)	(30.5)
	B	569	1,612	2.8	3.02	33.1
	C	298	3,162	10.6	2.24	44.7
	D	48	4,827	100.7	1.76	56.9
	E	--	5,182	All soilage	1.68	59.5
	F	--	"	"	"	"
	G	--	"	"	"	"
	H	--	"	"	"	"
200 pounds of gain:	A	(656)	(1,313)	(2.0)[f]	(3.18)	(62.8)
	B	(")	(")	(")[f]	(")	(")
	C	535	2,010	3.8	2.70	74.0
	D	418	2,786	6.7	2.32	86.1
	E	316	3,612	11.5	2.02	98.9
	F	228	4,482	19.6	1.78	112.6
	G	160	5,391	33.7	1.57	127.4
	H	114	6,330	55.8	1.40	143.3[g]
300 pounds of gain:	A	(683)	(1,365)	(2.0)[f]	(3.08)	(97.5)
	B	(")	(")	(")[f]	(")	(")
	C	634	1,648	2.6	2.86	104.8
	D	563	2,125	3.8	2.57	116.7
	E	499	2,633	5.3	2.32	129.4
	F	446	3,168	7.1	2.10	142.8[g]
	G	404	3,726	9.2	1.91	157.2[g]
	H	--	--	--	--	--[h]
350 pounds of gain:	A	(697)	(1,395)	(2.0)[f]	(3.02)	(115.9)
	B	(")	(")	(")[f]	(")	(")
	C	671	1,554	2.3	2.90	120.8
	D	612	1,944	[illegible]	[illegible]	[illegible]

... pounds of gain:	A	(697)	(1,395)	(2.0)[f]	(3.02)	(115.9)
	B	(")	(")	(")[f]	(")	(")
	C	671	1,554	2.3	2.90	120.8
	D	612	1,944	3.2	2.64	132.7
	E	560	2,359	4.2	2.41	145.3[g]
	F	517	2,797	5.4	2.21	158.5[g]
	G	--	--	--	--	--[h]
400 pounds of gain:	A	(714)	(1,427)	(2.0)[f]	(2.96)	(135.2)
	B	(")	(")	(")[f]	(")	(")
	C	704	1,489	2.1	2.91	137.4[g]
	D	655	1,813	2.8	2.68	149.2[g]
	E	--	--	--	--	--[h]
	F	--	--	--	--	--[h]
	G	--	--	--	--	--[h]
	H	--	--	--	--	--[h]

[a]Temperature is held constant at the overall mean.

[b]In addition to the corn and soilage there would also be fed a certain amount of the supplement shown in Table 3 at the rate of .2 of a pound per day.

[c]Ration is the ratio of soilage to corn.

[d]Average daily gain is determined by dividing total gain by number of days.

[e]The time equation, equation 39, presented in a later section provides the basis for these estimates.

[f]The highest concentrate ration that was fed during the experiment has been substituted whenever the predicted feed requirements resulted in a soilage-corn ratio of less than 2:1.

[g]The estimated feeding period exceeds the 136 day average feeding period in the experiment.

[h]Requires a feeding period in excess of 160 days.

Table 19. Least-cost rations for 850 pound good-to-choice feeder steers in terms of feed per 100 pounds of gain (without stilbestrol)[a]

	Price ratio area	Lbs. corn[b]	Lbs. soilage[b]	Ration[c]	Average daily gain[d]	Number of days[e]
100 pounds of gain:	A	(556)	(1,111)	(2.0)[f]	(3.45)	(29.0)
	B	(")	(")	(")[f]	(")	(")
	C	(")	(")	(")[f]	(")	(")
	D	(")	(")	(")[f]	(")	(")
	E	--	5,526	All soilage	1.90	52.6
	F	--	"	"	"	"
	G	--	"	"	"	"
	H	--	"	"	"	"
200 pounds of gain:	A	(604)	(1,208)	(2.0)[f]	(3.24)	(61.7)
	B	(")	(")	(")[f]	(")	(")
	C	(")	(")	(")[f]	(")	(")
	D	(")	(")	(")[f]	(")	(")
	E	463	2,396	5.2	2.67	75.0
	F	256	4,462	17.5	2.00	99.9
	G	126	6,188	49.4	1.63	122.6
	H	55	7,581	137.7	1.40	142.7[g]
300 pounds of gain:	A	(680)	(1,359)	(2.0)[f]	(2.96)	(101.5)
	B	(")	(")	(")[f]	(")	(")
	C	(")	(")	(")[f]	(")	(")
	D	(")	(")	(")[f]	(")	(")
	E	(")	(")	(")[f]	(")	(")
	F	608	2,099	3.5	2.60	115.4
	G	540	3,003	5.6	2.24	133.7
	H	504	3,731	7.4	2.00	149.7[g]

	E	(")	(")	(")[f]	(")	(")
	F	608	2,099	3.5	2.60	115.4
	G	540	3,003	5.6	2.24	133.7
	H	504	3,731	7.4	2.00	149.7[g]
350 pounds of gain:	A	(866)	(1,732)	(2.0)[f]	(2.75)	(127.1)
	B	(")	(")	(")[f]	(")	(")
	C	(")	(")	(")[f]	(")	(")
	D	(")	(")	(")[f]	(")	(")
	E	(")	(")	(")[f]	(")	(")
	F	(")	(")	(")[f]	(")	(")
	G	824	2,304	2.8	2.52	139.0[g]
	H	794	2,908	3.7	2.30	152.4[g]

[a]Temperature is held constant at the overall mean.

[b]In addition to the corn and soilage there would also be fed a certain amount of the supplement shown in Table 3 at the rate of .2 of a pound per day.

[c]Ration is the ratio of soilage to corn.

[d]Average daily gain is determined by dividing total gain by number of days.

[e]The time equation, equation 40, presented in a later section provides the basis for these estimates.

[f]The highest concentrate ration that was fed during the experiment has been substituted whenever the predicted feed requirements resulted in a soilage-corn ratio of less than 2:1.

[g]The estimated feeding period exceeds the 138 day average feeding period in the experiment.

100.7:1 including 48 pounds of corn and 4,827 pounds of soilage. The feeding period will be 57 days* long with an average daily gain of 1.76 pounds.

The rations in Table 19, the non-stilbestrol rations, are interpreted in a similar manner. The feeds -- corn and soilage -- reported in Tables 18 and 19 are in pounds of feed required per 100 pounds of gain. This method of stating feed requirements is consistent with the general practice followed in the animal sciences.

Estimation of the Time Function

The proportion of corn in the soilage-corn rations will affect the rate of gain as well as the cost of gain. For the ration that produces the fastest gains need not necessarily coincide with the least-cost ration.

In order to estimate what effects the different feed rations have on the rate of gain, a function that expresses the quantity of soilage consumed as a function of the quantity of corn fed and time was computed from the basic experimental data. The function, $F = f(C, T)$, where F denotes pounds of soilage, C denotes pounds of corn and T denotes time in days, can be used directly to derive iso-time curves. That is, for any given time this function can be used to

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*The time equation, equation 39, presented in a later section provides the basis for the time estimate.

determine the quantities of corn and soilage that will be consumed when these two feeds are fed in various proportions. The total time required to consume given quantities of corn and soilage may be obtained by solving the function, $F = f(C,T)$, for time. The function then expresses time as a function of corn and soilage, (i.e., $T = t(C,F)$).

The function that was used to predict soilage consumption was a quadratic function of the type:

$$(30)\quad F_t = a_1C_t + a_2T_t + a_3C_t^2 + a_4T_5^2 + a_5CT_t + u_t$$

where F refers to pounds of soilage consumed, C refers to pounds of corn consumed, T refers to time in days, the a_i's ($i = 1, \ldots, 5$) are constants to be estimated, u is a random variable and t is an index of time. The function is estimated without a constant term under the assumption that when corn consumption and time are both zero then forage consumption will also be zero. A further assumption was made, in that the random variable u_t was generated by the autoregressive scheme:

$$(31)\qquad u_t = \beta u_{t-1} + a_6H_t + e_t$$

where β is the autocorrelation coefficient, H is a temperature variable, a is a constant to be estimated and e_t is a random variable with the properties given by the equations in 13.

The temperature variable has been included in equation 31 under the assumption that the temperature during any one observation interval may increase or decrease the quantity of forage that would be consumed during that feeding interval.

Equation 30 can be rewritten for u_{t-1} to give:

$$(32) \quad F_{t-1} = a_1C_{t-1} + a_2T_{t-1} + a_3C^2_{t-1} + a_4T^2_{t-1} + a_5CT_{t-1} + u_{t-1}.$$

Equation 32 can now be solved for u_{t-1} and substituted into equation 31 to give:

$$(33) \quad u_t = \beta(F_{t-1} - a_1C_{t-1} - a_2T_{t-1} - a_3C^2_{t-1} - a_4T^2_{t-1} - a_5CT_{t-1}) + a_6H_t + e_t.$$

Now by substituting equation 33 into equation 30 and collecting terms, the following equation is obtained:

$$(34) \quad (F_t - \beta F_{t-1}) = a_1(C_t - \beta C_{t-1}) + a_2(T_t - \beta T_{t-1}) + a_3(C^2_t - \beta C^2_{t-1}) + a_4(T^2_t - \beta T^2_{t-1}) + a_5(CT_t - \beta CT_{t-1}) + a_6H_t + e_t.$$

Thus, if the variables in equation 30 are replaced by the transformed variables in equation 34, then the errors, e_t, are not autocorrelated and the least squares method of estimation will apply.

The autocorrelation coefficient that was used in the soilage consumption transformation equation was the same autocorrelation coefficient that was used in transforming the data for the production function. The same autocorrelation coefficient was used for two reasons. First, when the autocorrelation coefficient for the soilage consumption function was estimated in the same manner as for the production function, this method gave a biased estimate of β. For the experimental data show that there was a tendency to feed all lots, that were fed the same ration, the same quantities of corn and soilage. Some of the lots that were fed the same ration were actually fed the same quantities of corn and soilage for the entire feeding period. However, other lots, that were fed the same rations, may have been fed the same quantities of soilage and corn for a portion of the feeding period or at least up until it was quite evident that one of the two lots would actually eat more soilage and corn than the other lot. Only then would there be a definite difference in the quantities of feed fed to each lot and the differences were always in the same direction. Thus, the autocorrelation coefficient tends to be biased upward due to the tendency of feeding all lots, that were fed the same ration, the same quantities of corn and soilage. Second, since the data had already been transformed for the production function it was decided to use this transformed data in estimating the soilage consumption functions.

The overall soilage consumption functions estimated using the transformed data in the quadratic function are:

I. The overall stilbestrol function

$$(35)\quad F = -1.992155C + 82.750048T + .00026539C^2 + .07410081T^2 - .00856894CT + .88080396H$$

II. The overall non-stilbestrol function

$$(36)\quad F = -3.4605222C + 102.8699400T + .000094470C^2 + .04277036T^2 - .00066794CT + 17.99069800H.$$

The coefficient of determination, standard errors and the "t" values for the overall stilbestrol and non-stilbestrol soilage consumption functions are presented in Tables 20 and 21, respectively. The coefficient of determination is quite high for both the stilbestrol and non-stilbestrol functions. This high coefficient of determination indicates that a major portion of the variance in soilage consumption has been explained by the quadratic function. Most of the variables that were used in the regression are acceptable at a very high level of significance. Even though certain variables are acceptable only at a rather low level of probability, they have been retained in the regression since the basis for including the variables in the regression appeared to be consistent with nutrition and production logic.

The temperature coefficient for both the stilbestrol and non-stilbestrol soilage consumption function must be interpreted in light of the experimental feeding period which was started the second week in May and continued through to the latter part of September. When feeder cattle are first put

Table 20. Coefficient of determination, standard errors and "t" values for the overall stilbestrol soilage consumption function (equation 35)

R^2	Independent variable	Standard error of regression coefficient	"t" value	Level of significance
.9954	C	.30406658	6.552	$p < .001$
	T	3.04666905	27.161	$p < .001$
	C^2	.00013424	1.977	$.05 < p < .10$
	T^2	.02216920	3.343	$.001 < p < .005$
	CT	.00335280	2.556	$.01 < p < .025$
	H	4.41571104	.199	$p > .50$

Table 21. Coefficient of determination, standard errors and "t" values for the overall non-stilbestrol soilage consumption function (equation 36)

R^2	Independent variable	Standard error of regression coefficient	"t" value	Level of significance
.9947	C	.28670829	12.070	$p < .001$
	T	2.96349518	34.712	$p < .001$
	C^2	.00013380	.706	$p > .40$
	T^2	.02100461	2.036	$.05 < p < .025$
	CT	.00327306	.204	$p > .50$
	H	4.00302548	4.494	$p < .001$

on a new feeding ration that differs from what they were normally being fed, a certain period of time is required for the feeder cattle to adjust themselves to the new ration after which they will tend to consume larger quantities of the given ration. Furthermore, as the feeder cattle put on more weight they will also tend to eat more of the given ration. Therefore, it is not surprising that the coefficient for the temperature variable would have a positive sign since temperature is positively correlated with the conditions on consumption mentioned above. While the coefficient on the temperature variable for the stilbestrol function is not significant at the usual probability levels, it has, however, been retained in the consumption function in order to be consistent with both the stilbestrol and non-stilbestrol consumption functions.

The soilage consumption equations or the iso-time equations (i.e., equations 35 and 36) express the quantity of soilage (F) that will be consumed as a function of corn (C) and time (T). If time is held constant at a given number of days, then the soilage consumption equations will specify all possible combinations of soilage and corn that will be consumed within this given time period. Since the feeder steers have been on full feed, the iso-time function will predict the "stomach capacity" of the feeder steers for any given feeding period.

The slope of the iso-time curve or the "stomach capacity" curve indicates the substitution rate between feeds when time is held constant. That is, the substitution rate with time held constant indicates the amount one feed must be decreased, in order to increase the consumption of the other feed by one unit. The rate of substitution between soilage and corn for a given time period can be derived from the soilage consumption functions. The equations for predicting the rate at which corn substitutes for soilage in consumption for any given feeding period are:

I. With stilbestrol

(37) $$\frac{\partial F}{\partial C} = -1.992155 + .14820162C - .00856894T$$

II. Without stilbestrol

(38) $$\frac{\partial F}{\partial C} = -3.4605222 + .00018094C - .00066794T.$$

The iso-time schedules and the associated rates of substitution have been derived for 30, 60, 90, 120, 130 and 140 feeding days and are presented in Tables 22 and 23 for the overall stilbestrol and non-stilbestrol functions, respectively. The iso-time schedules have been plotted in Figure 22 for the overall stilbestrol function and in Figure 23 for the overall non-stilbestrol function. As mentioned earlier the slope of the iso-time curves at any given point indicates the rate at which corn substitutes for soilage in consumption. The curvature of the iso-time curves, as indicated in both Figures 22 and 23, change very little, suggesting that

Table 22. Iso-time schedules showing quantities of various feed combinations[a] that could ... of corn for soilage in consumption for six different feeding intervals (with s...

Lbs. corn	30 days Lbs. soilage			60 days Lbs. soilage			90 days Lbs. soilage		
	Lbs. soilage	Ration[d]	$\frac{\partial F}{\partial C}$[c]	Lbs. soilage	Ration	$\frac{\partial F}{\partial C}$	Lbs. soilage	Ration	$\frac{\partial F}{\partial C}$
0	2,549	--[e]	2.25	5,232	--[e]	2.51	8,046	--[e]	2.76
100	2,327	23.3	2.20	4,984	49.8	2.45	7,774	77.7	2.71
200	2,110	10.5	2.14	4,741	23.8	2.40	7,506	37.5	2.66
300	1,898	6.3	2.09	4,504	15.0	2.35	7,243	24.1	2.60
400	1,692	4.2	2.04	4,272	10.7	2.29	6,985	17.5	2.55
500	1,491	3.0	1.98	4,045	8.1	2.24	6,732	13.5	2.50
600	1,295	2.2	1.93	3,824	6.4	2.19	6,485	10.8	2.44
700	1,105	1.6[f]	1.88	3,607	5.2	2.13	6,243	8.9	2.39
800				3,397	4.2	2.08	6,007	7.5	2.34
900				3,191	3.5	2.02	5,776	6.4	2.29
1,000				2,991	3.0	1.98	5,550	5.5	2.23
1,100				2,796	2.5	1.92	5,329	4.8	2.18
1,200				2,606	2.2	1.87	5,114	4.3	2.13
1,300				2,422	1.9[f]	1.82	4,904	3.8	2.07
1,400							4,699	3.4	2.02
1,500							4,500	3.0	1.97
1,600							4,306	2.7	1.91
1,700							4,117	2.4	1.86
1,800							3,934	2.2	1.81
1,900							3,755	2.0	1.75
2,000							3,583	1.8[f]	1.70
2,100									
2,200									
2,300									
2,400									
2,500									
2,600									
2,700									
2,800									
2,900									
3,000									

[a]For each of the feed combinations there would also be fed a certain amount of the su... of a pound per day.

[b]Temperature is held constant at the overall mean.

[c]Indicates the marginal rate of substitution of corn for soilage in consumption.

[d]Ration is the ratio of soilage to corn.

[e]The all soilage ration.

[f]All feed combinations at this point exceed the 2:1 ration and, hence, are outside th...

could possibly be fed and the marginal rate of substitution
ith stilbestrol)[b]

120 days			130 days			140 days		
Lbs. soilage	Ration	$\frac{\partial F}{\partial C}$	Lbs. soilage	Ration	$\frac{\partial F}{\partial C}$	Lbs. soilage	Ration	$\frac{\partial F}{\partial C}$
10,997	--[e]	3.02	12,010	--[e]	3.11	13,037	--[e]	3.19
10,697	10.70	2.97	11,701	117.0	3.05	12,721	127.2	3.14
10,404	52.0	2.91	11,399	57.0	3.00	12,410	62.0	3.09
10,115	33.7	2.86	11,102	37.0	2.95	12,104	40.3	3.03
9,831	24.6	2.81	10,810	27.0	2.89	11,803	29.5	2.98
9,553	19.1	2.76	10,523	21.0	2.84	11,508	23.0	2.93
9,280	15.5	2.70	10,242	17.1	2.79	11,218	18.7	2.87
9,013	12.9	2.65	9,966	14.2	2.73	10,933	15.6	2.82
8,751	10.9	2.60	9,695	12.1	2.68	10,654	13.3	2.77
8,494	9.4	2.54	9,429	10.5	2.63	10,380	11.5	2.71
8,242	8.2	2.49	9,169	9.2	2.58	10,111	10.1	2.66
7,996	7.3	2.44	8,914	8.1	2.52	9,848	9.0	2.61
7,755	6.5	2.38	8,665	7.2	2.47	9,589	8.0	2.55
7,519	5.8	2.33	8,420	6.5	2.42	9,336	7.2	2.50
7,289	5.2	2.28	8,181	5.8	2.36	9,089	6.9	2.45
7,064	4.7	2.22	7,948	5.3	2.31	8,847	5.9	2.40
6,843	4.3	2.17	7,719	4.8	2.26	8,610	5.4	2.34
6,629	3.9	2.12	7,496	4.4	2.20	8,378	4.9	2.29
6,420	3.6	2.07	7,279	4.0	2.15	8,152	4.5	2.24
6,216	3.3	2.01	7,066	3.7	2.10	7,931	4.2	2.18
6,018	3.0	1.96	6,859	3.4	2.04	7,715	3.9	2.13
5,825	2.8	1.91	6,657	3.2	1.99	7,505	3.6	2.08
5,637	2.6	1.85	6,461	2.9	1.94	7,300	3.3	2.02
5,454	2.4	1.80	6,270	2.7	1.89	7,100	3.1	1.97
5,277	2.2	1.75	6,084	2.5	1.82	6,906	2.9	1.92
5,105	2.0	1.69	5,903	2.4	1.78	6,717	2.7	1.86
4,933	1.9[f]	1.64	5,728	2.2	1.73	6,533	2.5	1.81
			5,558	2.1	1.67	6,354	2.4	1.76
			5,393	1.9[f]	1.62	6,181	2.2	1.71
						6,013	2.1	1.65
						5,850	2.0	1.60

he supplement shown in Table 3. This supplement would be fed at the rate of .2

de the limits of the experiment.

Table 23. Iso-time schedules showing quantities of various feed combinations[a] that could possibly b
of corn for soilage in consumption for six different feeding intervals (without stilbestr

	30 days			60 days			90 days			120	
Lbs. corn	Lbs. soilage	Ration[d]	$\frac{\partial F}{\partial C}$[c]	Lbs. soilage	Ration	$\frac{\partial F}{\partial C}$	Lbs. soilage	Ration	$\frac{\partial F}{\partial C}$	Lbs. soilage	R
0	3,125	--[e]	3.48	6,326	--[e]	3.50	9,605	--[e]	3.52	12,960	
100	2,777	27.8	3.46	5,977	59.8	3.48	9,254	92.5	3.50	12,607	1[illegible]
200	2,432	12.2	3.44	5,630	28.1	3.46	8,904	44.5	3.48	12,256	[illegible]
300	2,089	7.0	3.42	5,284	17.6	3.44	8,557	28.5	3.46	11,907	[illegible]
400	1,747	4.4	3.40	4,941	12.4	3.43	8,212	20.5	3.45	11,559	[illegible]
500	1,408	2.8	3.39	4,599	9.2	3.41	7,868	15.7	3.43	11,214	[illegible]
600	1,070	1.8[f]	3.37	4,260	7.1	3.39	7,526	12.5	3.41	10,870	1[illegible]
700				3,922	5.6	3.37	7,187	10.3	3.39	10,528	1[illegible]
800				3,586	4.5	3.35	6,849	8.6	3.37	10,188	1[illegible]
900				3,252	3.6	3.33	6,516	7.2	3.35	9,850	1[illegible]
1,000				2,920	2.9	3.31	6,179	6.2	3.33	9,514	
1,100				2,590	2.4	3.29	5,846	5.3	3.31	9,180	
1,200				2,261	1.9[f]	3.27	5,516	4.6	3.29	8,845	
1,300							5,187	4.0	3.28	8,517	
1,400							4,861	3.5	3.26	8,189	
1,500							4,536	3.0	3.24	7,862	
1,600							4,214	2.6	3.22	7,537	
1,700							3,893	2.3	3.20	7,214	
1,800							3,574	2.0	3.18	6,893	
1,900							3,257	1.7[f]	3.16	6,574	
2,000										6,257	
2,100										5,941	
2,200										5,628	
2,300										5,316	
2,400										5,007	
2,500										4,699	
2,600											
2,700											
2,800											
2,900											
3,000											

[a]For each of the feed combinations there would also be fed a certain amount of the supplement sh
of a pound per day.

[b]Temperature is held constant at the overall mean.

[c]Indicates the marginal rate of substitution of corn for soilage in consumption.

[d]Ration is the ratio of soilage to corn.

[e]The all soilage ration.

[f]All feed combinations at this point exceed the 2:1 ration and, hence, are outside the limits of

possibly be fed and the marginal rates of substitution
stilbestrol)[b]

120 days			130 days			140 days		
Lbs. scilage	Ration	$\frac{\partial F}{\partial C}$	Lbs. scilage	Ration	$\frac{\partial F}{\partial C}$	Lbs. scilage	Ration	$\frac{\partial F}{\partial C}$
12,960	--[e]	3.54	14,096	--[e]	3.55	15,240	--[e]	3.55
12,607	126.1	3.52	13,742	137.4	3.53	14,885	148.9	3.54
12,256	61.3	3.50	13,390	67.0	3.51	14,533	72.7	3.52
11,907	39.7	3.48	13,040	43.5	3.49	14,182	47.3	3.50
11,559	28.9	3.47	12,692	31.7	3.47	13,834	34.6	3.48
11,214	22.4	3.45	12,346	24.7	3.45	13,487	27.0	3.46
10,870	18.1	3.43	12,002	20.0	3.43	13,142	21.9	3.44
10,528	15.0	3.41	11,659	16.7	3.42	12,799	18.3	3.42
10,188	12.7	3.39	11,318	14.1	3.40	12,457	15.6	3.40
9,850	10.9	3.37	10,980	12.2	3.38	12,111	13.5	3.38
9,514	9.5	3.35	10,643	10.6	3.36	11,781	11.8	3.37
9,180	8.3	3.33	10,308	9.2	3.34	11,445	10.4	3.35
8,846	7.4	3.31	9,975	8.3	3.32	11,111	9.3	3.33
8,517	6.6	3.30	9,644	7.4	3.30	10,780	8.3	3.31
8,189	5.8	3.28	9,315	6.7	3.28	10,450	7.5	3.29
7,862	5.2	3.26	8,987	6.0	3.26	10,122	6.7	3.27
7,537	4.7	3.24	8,662	5.4	3.25	9,795	6.1	3.25
7,214	4.2	3.22	8,338	4.9	3.23	9,471	5.6	3.23
6,893	3.8	3.20	8,017	4.5	3.21	9,149	5.1	3.21
6,574	3.5	3.18	7,697	4.1	3.19	8,828	4.6	3.20
6,257	3.1	3.16	7,379	3.7	3.17	8,510	4.3	3.18
5,941	2.8	3.14	7,063	3.4	3.15	8,193	3.9	3.16
5,628	2.6	3.13	6,749	3.1	3.13	7,878	3.6	3.14
5,316	2.3	3.11	6,437	2.8	3.11	7,566	3.3	3.12
5,007	2.1	3.09	6,126	2.6	3.09	7,255	3.0	3.10
4,699	1.9[f]	3.07	5,818	2.3	3.08	6,945	2.8	3.08
			5,511	2.1	3.06	6,638	2.6	3.06
			5,207	1.9[f]	3.04	6,333	2.3	3.04
						6,029	2.2	3.03
						5,728	2.0	3.01
						5,428	1.8[f]	2.99

plement shown in Table 3. This supplement would be fed at the rate of .2

limits of the experiment.

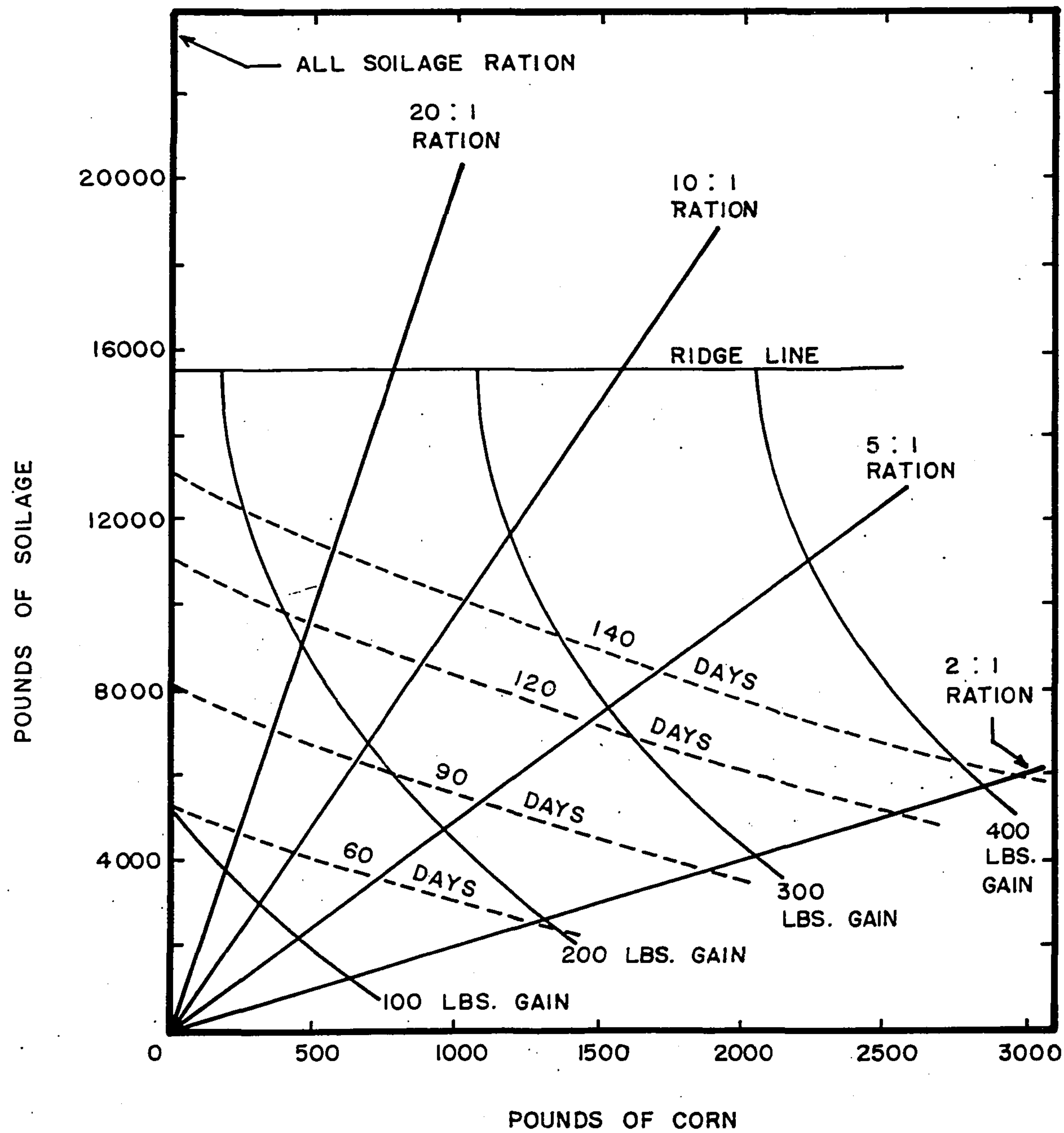

Figure 22. Iso-time curves, gain isoquants and selected ration lines for the overall stilbestrol function (temperature is held constant at the overall mean)

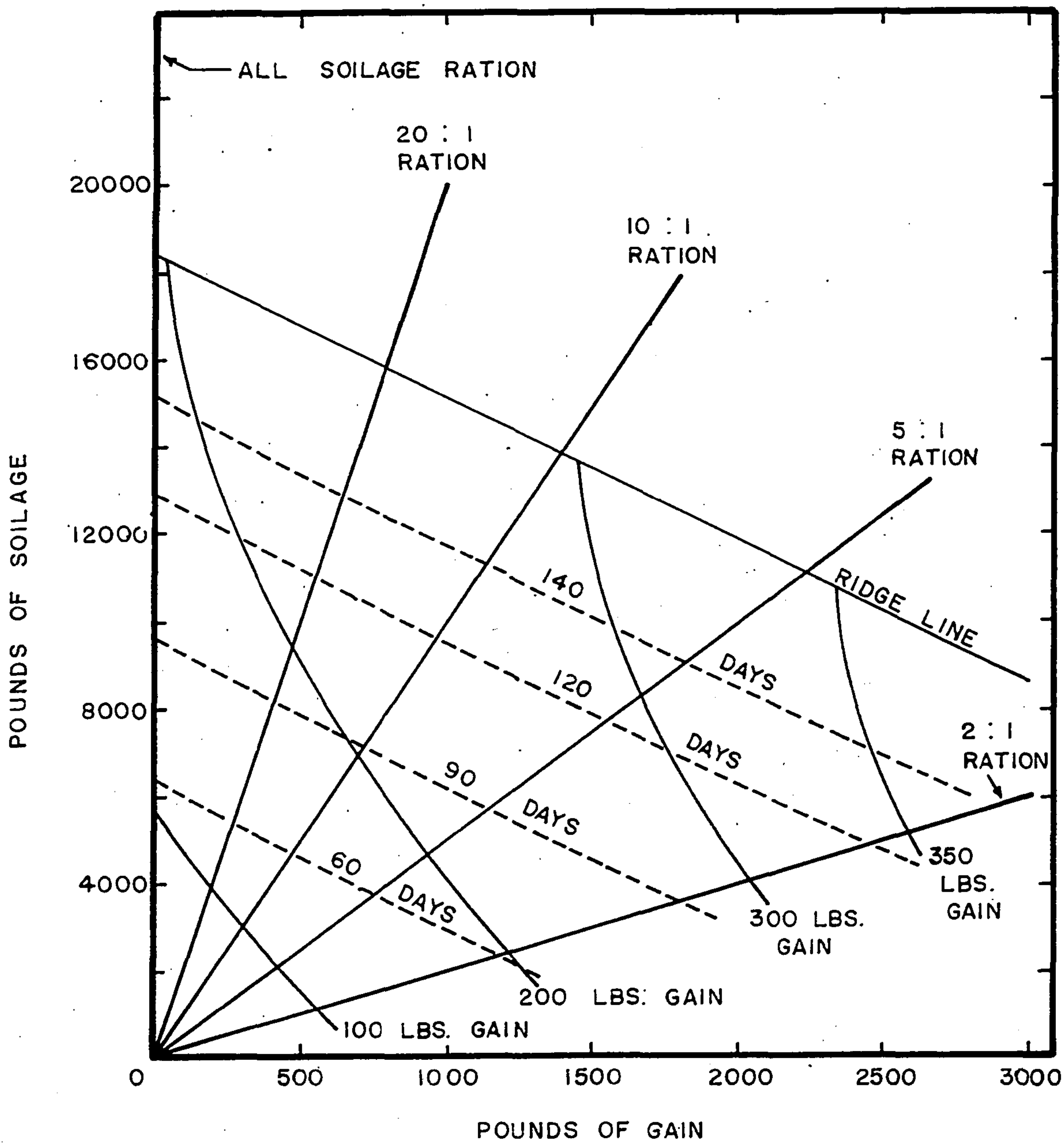

Figure 23. Iso-time curves, gain isoquants and selected ration lines for the overall non-stilbestrol function (temperature is held constant at the overall mean)

the substitution rates between the two feeds in consumption are nearly constant. The iso-time curves in both Figures 22 and 23 are slightly convex to the origin. However, if the range on the experimental rations had been extended past the 2:1 ration the iso-time curves or the "stomach capacity" curves may have been slightly concave to the origin, at least for the heavy corn rations. These results would have been consistent with the hypothesis presented in an earlier chapter.

By superimposing the gain isoquants on the iso-time curves, it is possible to get some idea of the portion of the production surface that is relevant for various feeding periods. Figures 22 and 23 show the predicted gain isoquants superimposed over the predicted iso-time curves for the stilbestrol and non-stilbestrol rations, respectively. For example, in both Figures 22 and 23, the all soilage ration line, the 140 day iso-time curve and the 2:1 ration line depict the boundary lines for the 140 day feeding period.

Time relationships

Equations that express the total time required to consume various quantities of soilage and corn may be derived from the overall soilage consumptions functions. Thus, the overall time equations for the stilbestrol and non-stilbestrol rations as derived from the two overall soilage consumption equations, 35 and 36, are as follows:

I. With stilbestrol

(39) $$T = -558.36128626 + .057819480C \pm 6.7475645\,(6{,}847.57044400 - .00000523C^2 - .827679190C - .26107315H + .29640324F)^{1/2}$$

II. Without stilbestrol

(40) $$T = -1{,}176.48647060 + .007638990C \pm 11.436640\,(10{,}582.22455560 - .00001571C^2 + .454609220C - 3.07787452H + .17108144F)^{1/2}.$$

The time equations (*i.e.*, equations 39 and 40) express the total time (T) required to consume a given quantity of corn and soilage as a function of the soilage (F) and corn (C) fed. Thus, it is possible to predict the time required to produce various levels of gain, when different soilage-corn rations are fed, by substituting into the time equation the predicted feed requirements for the various levels of gain. Table 24 shows, for a selected number of stilbestrol soilage-corn rations, the time required to produce various levels of gain.* In all cases, the time required to produce a given level of gain decreases as the proportion of corn in the ration increases. Too, the predicted values indicate that for a given feeding period the maximum level of gain is

*The predicted feed requirements for selected stilbestrol and non-stilbestrol rations for various levels of gain are shown in Tables 8 and 9.

Table 24. Predicted total time required to produce various levels of gain for eight select
(temperature is held constant at the overall mean)[a]

Selected rations[b]	100 lbs. gain			200 lbs. gain			300 lbs. gain		
	Lbs. corn	Lbs. soilage	Time (days)	Lbs. corn	Lbs. soilage	Time (days)	Lbs. corn	Lbs. soilage	Time (days)
All soilage	--	5,182	59.5						
20:1	192	3,834	49.6	451	9,028	113.1			
15:1	236	3,545	47.5	539	8,087	105.7			
10:1	309	3,091	44.2	680	6,799	95.6	1,178	11,777	162.4[c]
8:1	353	2,825	42.2	765	6,117	90.3	1,278	10,228	149.0[c]
5:1	451	2,256	38.0	953	4,766	79.8	1,528	7,639	127.2
3:1	557	1,672	33.6	1,160	3,480	69.7	1,821	5,463	109.1
2:1	634	1,268	30.5	1,313	2,626	62.8	2,049	4,095	97.5

[a]For each of the feed combinations there would also be fed a certain amount of the supp
would be fed at the rate of .2 of a pound per day.

[b]Ration is the ratio of soilage to corn.

[c]The estimated feeding period exceeds the 136 day average feeding period in the experim

Table 25. Predicted total time required to produce various levels of gain for eight selecte
(temperature is held constant at the overall mean)[a]

Selected rations[b]	100 lbs. gain			200 lbs. gain			300 lbs. gain		
	Lbs. corn	Lbs. soilage	Time (days)	Lbs. corn	Lbs. soilage	Time (days)	Lbs. corn	Lbs. soilage	Time (days)
All soilage	--	5,526	52.6						
20:1	195	3,896	43.7	470	9,401	102.9			
15:1	237	3,556	41.8	559	8,386	96.5			
10:1	303	3,031	39.0	696	6,956	87.8	1,474	12,445	159.5[c]
8:1	341	2,732	37.4	774	6,190	83.2	1,487	11,898	155.2[c]
5:1	422	2,111	34.2	937	4,687	74.4	1,666	8,328	128.9
3:1	502	1,507	31.0	1,100	3,299	66.6	1,883	5,650	111.2
2:1	556	1,111	29.0	1,208	2,417	61.7	2,039	4,078	101.5

[a]For each of the feed combinations there would also be fed a certain amount of the suppl
would be fed at the rate of .2 of a pound per day.

[b]The ration is defined as the ratio of soilage to corn.

[c]The estimated feeding period exceeds the 136 day average feeding period in the experime

vels of gain for eight selected stilbestrol soilage-corn rations

	300 lbs. gain			350 lbs. gain			400 lbs. gain		
)	Lbs. corn	Lbs. soilage	Time (days)	Lbs. corn	Lbs. soilage	Time (days)	Lbs. corn	Lbs. soilage	Time (days)
	1,178	11,777	162.4[c]						
	1,273	10,228	149.0[c]						
	1,528	7,639	127.2	1,855	9,275	154.0[c]	2,221	11,105	183.9[c]
	1,821	5,463	109.1	2,180	6,539	130.5	2,562	4,685	153.1[c]
	2,049	4,095	97.5	2,441	4,881	115.9	2,854	5,709	135.2

a certain amount of the supplement shown in Table 3. This supplement

feeding period in the experiment.

els of gain for eight selected non-stilbestrol soilage-corn rations

300 lbs. gain			350 lbs. gain		
Lbs. corn	Lbs. soilage	Time (days)	Lbs. corn	Lbs. soilage	Time (days)
1,474	12,445	159.5[c]			
1,487	11,898	155.2[c]			
1,666	8,328	128.9	2,354	9,646	159.7[c]
1,883	5,650	111.2	2,447	7,341	142.0[c]
2,039	4,078	101.5	2,597	5,195	127.1

a certain amount of the supplement shown in Table 3. This supplement

eeding period in the experiment.

attained with the heaviest corn ration. Table 25 shows the number of days required to produce various levels of gain for a selected number of non-stilbestrol rations.

The average daily rate of gain for various levels of gain are presented in Table 26 for a selected number of stilbestrol rations. The average daily rate of gain is found by dividing total gain by the number of days required to attain this gain. For any given level of gain, the average daily gain increases as the proportion of corn in the ration increases. For any given ration the average daily rate of gain diminishes as the beef animal takes on heavier weights. The estimates presented in Table 27 are for the non-stilbestrol rations and except for the magnitude of the average daily rates of gain they follow a pattern similar to those for the stilbestrol rations.

Time equations for selected rations

The overall time equations may be reduced to individual time equations for selected rations in the same manner as the overall production functions were reduced to individual gain equations. These individual time equations for selected rations are shown in Tables 28 and 29 for the stilbestrol and non-stilbestrol rations, respectively.

The estimated number of days required to feed various quantities of the selected rations are shown in Table 30 for the stilbestrol rations and in Table 31 for the non-stil-

Table 26. Average daily gains for various levels of gain when 850 pound good-to-choice feeder steers are fed selected stilbestrol soilage-corn rations (temperature is held constant at the overall mean)

Total pounds of gain	Average daily rate of gain for selected rations:[a] (in lbs.)							
	All soilage	20:1	15:1	10:1	8:1	5:1	3:1	2:1
50	1.818	2.101	2.183	2.336	2.439	2.688	3.030	3.333
100	1.681	2.016	2.105	2.262	2.370	2.632	2.976	3.279
150	1.472	1.908	2.011	2.183	2.297	2.573	2.924	3.233
200		1.768	1.892	2.092	2.215	2.506	2.869	3.185
250		1.556[b]	1.739	1.983	2.122	2.434	2.812	3.133
300			1.568[b]	1.847[b]	2.013[b]	2.358	2.750	3.077
350						2.273[b]	2.684	3.020
400							2.613[b]	2.959

[a]Ration is the ratio of soilage to corn.

[b]Indicates a feeding period of more than 136 days. See Table 24.

Table 27. Average daily gains for various levels of gain when 850 pound good-to-choice feeder steers are fed selected non-stilbestrol soilage-corn rations (temperature is held constant at the average mean)

Total pounds of gain	Average daily rate of gain for selected rations:[a] (in lbs.)							
	All soilage	20:1	15:1	10:1	8:1	5:1	3:1	2:1
50	2.058	2.415	2.513	2.674	2.778	3.030	3.311	3.546
100	1.901	2.288	2.392	2.564	2.674	2.924	3.226	3.448
150	1.685	2.140	2.249	2.435	2.551	2.814	3.119	3.356
200		1.944	2.073	2.278	2.404	2.688	3.003	3.241
250		1.598[b]	1.806	2.073	2.218	2.530	2.867	3.113
300				1.881[b]	1.933[b]	2.327	2.698	2.956
350						2.192[b]	2.465[b]	2.754
400								2.426[b]

[a]Ration is the ratio of soilage to corn.

[b]Indicates a feeding period of more than 138 days. See Table 25.

Table 28. Time equations, derived from the overall stilbestrol time function, to predict the time required for 850 pound good-to-choice feeder steers to consume various quantities of selected rations

Ration[a]	Prediction equations for time in days
Ration A All soilage	$T_A = -558.36128626 \pm 6.7475645\ (6{,}847.57044400 + .29640324\ \gamma_A{}^{b} - .26107315H)^{1/2}$
Ration B 20:1	$T_B = -558.36128626 + .00275331\ \gamma_B \pm 6.7475645\ (6{,}847.57044400 - .00000001\ \gamma_B^2 + .24287550\ \gamma_B - .26107315H)^{1/2}$
Ration C 15:1	$T_C = -558.36128626 + .00361372\ \gamma_C \pm 6.7475645\ (6{,}847.57044400 - .00000002\ \gamma_C^2 + .22614809\ \gamma_C - .26107315H)^{1/2}$
Ration D 10:1	$T_D = -558.36128626 + .00525632\ \gamma_D \pm 6.7475645\ (6{,}847.57044400 - .00000004\ \gamma_D^2 + .19421393\ \gamma_D - .26107315H)^{1/2}$
Ration E 8:1	$T_E = -558.36128626 + .00642439\ \gamma_E \pm 6.7475645\ (6{,}847.57044400 - .00000006\ \gamma_E^2 + .17150520\ \gamma_E - .26107315H)^{1/2}$
Ration F 5:1	$T_F = -558.36128626 + .00963658\ \gamma_F \pm 6.7475645\ (6{,}847.57044400 - .00000014\ \gamma_F^2 + .10905617\ \gamma_F - .26107315H)^{1/2}$
Ration G 3:1	$T_G = -558.36128626 + .01445487\ \gamma_G \pm 6.7475645\ (6{,}847.57044400 - .00000033\ \gamma_G^2 + .01538263\ \gamma_G - .26107315H)^{1/2}$
Ration H 2:1	$T_H = -558.36128626 + .01927316\ \gamma_H \pm 6.7475645\ (6{,}847.57044400 - .00000058\ \gamma_H^2 - .07829090\ \gamma_H - .26107315H)^{1/2}$

[a]Ration is the ratio of soilage to corn.

[b]γ denotes total pounds of feed of the particular ration.

Table 29. Time equations, derived from the overal non-stilbestrol time function, to predict the time required for 850 pound good-to-choice feeder steers to consume various quantities of selected rations

Ration[a]	Prediction equations for time in days
Ration A All soilage	$T_A = -1{,}176.48647060 \pm 11.436640\ (10{,}582.22455560 + .17108144\ \gamma_A{}^{b} - 3.07787452H)^{1/2}$
Ration B 20:1	$T_B = -1{,}176.48647060 + .00036376\ \gamma_B \pm 11.436640\ (10{,}582.22455560 - .00000004\ \gamma_B^2 + .18458276\ \gamma_B - 3.07787452H)^{1/2}$
Ration C 15:1	$T_C = -1{,}176.48647060 + .00047744\ \gamma_C \pm 11.436640\ (10{,}582.22455560 - .00000006\ \gamma_C^2 + .18880193\ \gamma_C - 3.07787452H)^{1/2}$
Ration D 10:1	$T_D = -1{,}176.48647060 + .00069445\ \gamma_D \pm 11.436640\ (10{,}582.22455560 - .00000013\ \gamma_D^2 + .19685669\ \ _D - 3.07787452H)^{1/2}$
Ration E 8:1	$T_E = -1{,}176.4864706 + .00084878\ \gamma_E \pm 11.436640\ (10{,}582.22455560 - .00000019\ \gamma_E^2 + .20258453\ \gamma_E - 3.07787452H)^{1/2}$
Ration F 5:1	$T_F = -1{,}176.4864706 + .00127317\ \gamma_F \pm 11.436640\ (10{,}582.22455560 - .00000044\ \gamma_F^2 + .21833607\ \gamma_F - 3.07787452H)^{1/2}$
Ration G 3:1	$T_G = -1{,}176.4864706 + .00190975\ \gamma_G \pm 11.436640\ (10{,}582.22455560 - .00000098\ \gamma_G^2 + .24196339\ \gamma_G - 3.07787452H)^{1/2}$
Ration H 2:1	$T_H = -1{,}176.4864706 + .00254633\ \gamma_H \pm 11.436640\ (10{,}582.22455560 - .00000175\ \gamma_H^2 + .26559070\ \gamma_H - 3.07787452H)^{1/2}$

[a]Ration is the ratio of soilage to corn.

[b]γ denotes total pounds of feed of the particular ration.

Table 30. Estimated total time required for 850 pound good-to-choice feeder steers to consume various amounts of selected soilage-corn rations (with stilbestrol)[a]

Pounds of feed fed	Total days[b] required to feed various quantities of selected rations:[c] All soilage	20:1	15:1	10:1	8:1	5:1	3:1	2:1
500	6.0	6.3	6.4	6.6	6.7	7.0	7.5	8.0
1,000	12.0	12.6	12.8	13.1	13.4	14.1	15.1	16.0
1,500	17.8	18.8	19.1	19.6	20.0	21.1	22.6	24.0 d
2,000	23.7	25.0	25.4	26.1	26.7	28.1	30.1	32.0
2,500	29.4	31.1	31.6	32.6	33.3	35.1	37.6	40.0
3,000	35.1	37.2	37.8	39.0	39.8	42.0	45.1	47.9
3,500	40.8	43.3	44.0	45.4	46.4	49.0	52.6	55.9 e
4,000	46.4	49.3	50.2	51.8	53.0	56.0	60.1	63.8
4,500	52.0	55.3	56.3	58.2	59.5	62.9	67.6	71.7
5,000	57.5	61.2	62.4	64.5	66.0	69.8	75.1	79.6
5,500	62.9	67.2	68.4	70.8	72.5	76.8	82.5	87.4
6,000	68.3	73.1	74.5	77.1	78.9	83.7	90.0	95.3 f
6,500	73.7	78.9	80.5	83.4	85.4	90.6	97.5	103.1
7,000	79.0	84.7	86.4	89.6	91.8	97.5	104.9	110.9
7,500	84.3	90.5	92.4	95.9	98.2	104.4	112.4	118.7
8,000	89.5	96.3	98.3	102.1	104.6	111.2	119.8	126.4
8,500	94.7	102.0	104.2	108.2	111.0	118.1	127.2	134.2 g
9,000	99.8	107.7	110.1	114.4	117.4	125.0	134.6	141.9[h]
9,500	104.9	113.4	115.9	120.5	123.7	131.8	142.0[h]	149.6
10,000	110.0	119.0	121.7	126.7	130.0	138.6[h]	149.5	157.3
10,500	115.0	124.6	127.5	132.8	136.3[h]	145.5	156.9	165.0
11,000	120.0	130.2	133.2	138.8[h]	142.6	152.3	164.3	
11,500	125.0	135.8	139.0[h]	144.9	148.9	159.1		
12,000	129.9	141.3[h]	144.7	150.9	155.2	165.9		
12,500	134.8	146.8	150.4	157.0	161.4			
13,000	139.6[h]	152.3	156.0	163.0				
13,500	144.5	157.7	161.7					
14,000	149.2							
14,500	154.0							
15,000	158.7							

9,000	99.8	107.7	110.1	114.4	117.4	125.0	134.6	141.9[h]
9,500	104.9	113.4	115.9	120.5	123.7	131.8	142.0[h]	149.6
10,000	110.0	119.0	121.7	126.7	130.0	138.6[h]	149.5	157.3
10,500	115.0	124.6	127.5	132.8	136.3[h]	145.5	156.9	165.0
11,000	120.0	130.2	133.2	138.8[h]	142.6	152.3	164.3	
11,500	125.0	135.8	139.0[h]	144.9	148.9	159.1		
12,000	129.9	141.3[h]	144.7	150.9	155.2	165.9		
12,500	134.8	146.8	150.4	157.0	161.4			
13,000	139.6[h]	152.3	156.0	163.0				
13,500	144.5	157.7	161.7					
14,000	149.2							
14,500	154.0							
15,000	158.7							

[a]Temperature is held constant at the overall mean.

[b]All values are derived from the equations in Table 28.

[c]Ration is the ratio of soilage to corn.

[d]The horizontal line indicates 100 lbs. of gain (see Table 11).

[e]The horizontal line indicates 200 lbs. of gain (see Table 11).

[f]The horizontal line indicates 300 lbs. of gain (see Table 11).

[g]The horizontal line indicates 400 lbs. of gain (see Table 11).

[h]Indicates a feeding period of more than 136 days.

Table 31. Estimated total time required for 850 pound good-to-choice feeder steers to consume various amounts of selected soilage-corn rations (without stilbestrol)[a]

Pounds of feed fed	Total days[b] required to feed various quantities of selected rations:[c] All soilage	20:1	15:1	10:1	8:1	5:1	3:1	2:1
500	4.9	5.4	5.6	5.9	6.2	6.8	7.8	8.8
1,000	9.7	10.8	11.2	11.8	12.3	13.6	15.6	17.5
1,500	14.5	16.2	16.7	17.7	18.4	20.4	23.3	26.1 [d]
2,000	19.3	21.5	22.2	23.6	24.5	27.1	30.9	34.6
2,500	24.1	26.9	27.7	29.4	30.5	33.7	38.4	43.1
3,000	28.8	32.2	33.2	35.2	36.6	40.3	45.9	51.4
3,500	33.6	37.4	38.6	40.9	42.5	46.9	53.4	59.7 [e]
4,000	38.3	42.7	44.1	46.7	48.5	53.5	60.7	67.8
4,500	43.0	47.9	49.5	52.4	54.4	60.0	68.1	75.9
5,000	47.7	53.2	54.9	58.1	60.3	66.4	75.3	83.9
5,500	52.3	58.4	60.2	63.7	66.2	72.8	82.5	91.8
6,000	57.0	63.5	65.5	69.3	72.0	79.2	89.7	99.7 [f]
6,500	61.6	68.7	70.9	75.0	77.8	85.6	96.8	107.4
7,000	66.2	73.8	76.1	80.5	83.6	91.9	103.8	115.1
7,500	70.8	78.9	81.4	86.1	89.4	98.2	110.8	122.7 [g]
8,000	75.4	84.0	86.7	91.6	95.1	104.4	117.7	130.2
8,500	80.0	89.1	91.9	97.1	100.8	110.6	124.6	137.7
9,000	84.5	94.1	97.1	102.6	106.5	116.8	131.4	145.1[h]
9,500	89.1	99.2	102.3	108.1	112.1	122.9	138.2[h]	152.4
10,000	93.6	104.2	107.4	113.5	117.7	129.0	144.9	159.6 [i]
10,500	98.1	109.2	112.6	118.9	123.3	135.1	151.6	166.8
11,000	102.6	114.2	117.7	124.3	128.9	141.1[h]	158.2	
11,500	107.0	119.1	122.8	129.7	134.5	147.1	164.8	
12,000	111.5	124.1	127.9	135.0	140.0[h]	153.1		
12,500	115.9	129.0	133.0	140.4[h]	145.5	159.0		
13,000	120.4	133.9	138.0	145.7	150.9			
13,500	124.8	138.8[h]	143.0[h]	150.9	156.4			
14,000	129.2	143.7	148.0	156.2				
14,500	133.5	148.5	153.0					
15,000	137.9	153.4	158.0					

10,000	93.6	104.2	107.4	113.5	117.7	129.0	144.9	159.6[i]
10,500	98.1	109.2	112.6	118.9	123.3	135.1	151.6	166.8
11,000	102.6	114.2	117.7	124.3	128.9	141.1[h]	158.2	
11,500	107.0	119.1	122.8	129.7	134.5	147.1	164.8	
12,000	111.5	124.1	127.9	135.0	140.0[h]	153.1		
12,500	115.9	129.0	133.0	140.4[h]	145.5	159.0		
13,000	120.4	133.9	138.0	145.7	150.9			
13,500	124.8	138.8[h]	143.0[h]	150.9	156.4			
14,000	129.2	143.7	148.0	156.2				
14,500	133.5	148.5	153.0					
15,000	137.9	153.4	158.0					
15,500	142.3[h]	158.2						
16,000	146.6							
16,500	150.9							
17,000	155.2							

[a]Temperature is held constant at the overall mean.

[b]All values are derived from the equations in Table 29.

[c]Ration is the ratio of soilage to corn.

[d]The horizontal line indicates 100 pounds of gain (see Table 13).

[e]The horizontal line indicates 200 pounds of gain (see Table 13).

[f]The horizontal line indicates 300 pounds of gain (see Table 13).

[g]The horizontal line indicates 350 pounds of gain (see Table 13).

[h]Indicates a feeding period of more than 136 days.

[i]The horizontal line indicates 400 pounds of gain (see Table 13).

bestrol rations. The predicted values show that the time required to consume any given quantity of feed increases as the proportion of corn in the ration increases. For example, in Table 30, 5,000 pounds of the all soilage ration will be consumed by a feeder steer in 57.5 days, whereas, 61.2 days are required to consume 5,000 pounds of a 20:1 ration, 62.4 days for a 15:1 ration, 64.5 days for a 10:1 ration, etc.

Since fresh chopped pasture forage has a very high moisture content and is highly palatable, a feeder steer is able to consume and digest, in any given time period, larger quantities of the all soilage ration than rations that contain successively greater proportions of corn which has a relatively low moisture content. Even though a feeder steer will consume greater quantities of the all soilage ration than rations that contain successively greater proportions of corn, the total digestible nutrient intake will be less for the all soilage ration than for rations containing successively greater proportions of corn. This relationship is implied when Tables 12 and 30, for the stilbestrol rations, and Tables 13 and 31, for the non-stilbestrol rations, are compared. In Tables 12 and 13 time lines have been drawn across the various ration columns to indicate feeding periods of equal length. Thus, Tables 12 and 30 and Tables 13 and 31, for the stilbestrol and non-stilbestrol rations, show that, for any given time period, total gains are greater for the heavier corn rations. Furthermore, this greater gain is attained by feeding less total pounds of feed than was fed for any other ration for the same period of time.

The average daily gains from feeding various quantities of selected rations are shown in Table 32 for the stilbestrol rations and in Table 33 for the non-stilbestrol rations. The average daily gains are found by dividing the predicted total gain by the predicted time required to attain this gain. The average daily gains in Table 32, for the stilbestrol rations, are found by dividing the predicted total gains in Table 12 by the corresponding predicted total time in Table 30. The average daily gains for the non-stilbestrol rations are shown in Table 33 and are derived in a similar manner by dividing the total gains in Table 13 by the corresponding total time in Table 31. The average daily gains, for example, in Table 32, increase for any given level of feed consumption as the proportion of corn in the ration increases. However, for any given ration, the average daily gain decreases as the quantity of feed fed increases. These results are expected, since for any one ration a decrease in the daily gain, as greater quantities of feed are fed, is consistent with diminishing returns to feed.

Tables 34 and 35 are included to show for selected rations the predicted quantities of corn and soilage that would be fed and the associated gains for various feeding

Table 32. Average daily gains from feeding various quantities of selected soilage-corn rations to 850 pound good-to-choice feeder steers (with stilbestrol)[a]

Pounds of feed fed	Average daily gains in pounds for selected rations:[b] All soilage	20:1	15:1	10:1	8:1	5:1	3:1	2:1
500	1.90	2.16	2.23	2.38	2.48	2.74	3.08	3.36
1,000	1.87	2.13	2.21	2.37	2.46	2.70	3.03	3.34
1,500	1.86	2.12	2.20	2.35	2.44	2.68	3.01	3.31
2,000	1.83	2.10	2.18	2.33	2.41	2.66	2.99	3.28
2,500	1.81	2.08	2.16	2.30	2.39	2.64	2.97	3.25
3,000	1.79	2.06	2.14	2.28	2.38	2.62	2.95	3.23
3,500	1.76	2.04	2.12	2.26	2.36	2.60	2.92	3.20
4,000	1.74	2.02	2.10	2.24	2.33	2.58	2.90	3.18
4,500	1.71	1.99	2.07	2.22	2.31	2.56	2.88	3.16
5,000	1.69	1.98	2.05	2.20	2.29	2.54	2.85	3.13
5,500	1.67	1.95	2.03	2.18	2.27	2.51	2.83	3.11
6,000	1.64	1.93	2.01	2.16	2.25	2.49	2.81	3.08
6,500	1.61	1.91	1.99	2.13	2.23	2.47	2.78	3.06
7,000	1.59	1.88	1.97	2.11	2.21	2.45	2.76	3.04
7,500	1.56	1.86	1.94	2.09	2.19	2.43	2.74	3.01
8,000	1.54	1.84	1.92	2.07	2.17	2.41	2.72	2.99
8,500	1.51	1.81	1.90	2.05	2.15	2.39	2.69	2.96
9,000	1.48	1.79	1.87	2.03	2.12	2.36	2.67	2.94[c]
9,500	1.46	1.77	1.85	2.00	2.10	2.34	2.65[c]	2.91
10,000	1.43	1.74	1.83	1.98	2.08	2.32[c]	2.62	2.89
10,500	1.40	1.72	1.81	1.96	2.06[c]	2.30	2.60	2.86
11,000	1.37	1.69	1.78	1.94[c]	2.04	2.28	2.58	
11,500	1.34	1.67	1.76[c]	1.91	2.01	2.26		
12,000	1.31	1.64[c]	1.73	1.89	1.99	2.23		
12,500	1.28	1.62	1.71	1.87	1.97			
13,000	1.25[c]	1.59	1.69	1.84				
13,500	1.22	1.57	1.66					
14,000	1.19							
14,500	1.16							
15,000	1.13							

[a]Temperature is held constant at the overall mean.

[b]Ration is the ratio of soilage to corn.

[c]Indicates a feeding period of more than 136 days. See Table 30.

Table 33. Average daily gains from feeding various quantities of selected soilage-corn rations to 850 pound good-to-choice feeder steers (without stilbestrol)[a]

Pounds of feed fed	Average daily gains in pounds for selected rations:[b] All soilage	20:1	15:1	10:1	8:1	5:1	3:1	2:1
500	2.14	2.50	2.59	2.76	2.82	3.09	3.36	3.57
1,000	2.13	2.47	2.54	2.72	2.81	3.04	3.31	3.52
1,500	2.11	2.44	2.53	2.68	2.78	3.00	3.26	3.47
2,000	2.09	2.41	2.50	2.64	2.74	2.96	3.22	3.42
2,500	2.06	2.38	2.47	2.61	2.71	2.93	3.18	3.36
3,000	2.04	2.35	2.44	2.58	2.67	2.90	3.14	3.31
3,500	2.01	2.33	2.41	2.55	2.64	2.86	3.09	3.25
4,000	1.98	2.30	2.38	2.52	2.61	2.81	3.04	3.20
4,500	1.96	2.27	2.35	2.49	2.58	2.77	2.99	3.14
5,000	1.93	2.23	2.32	2.45	2.54	2.74	2.95	3.09
5,500	1.90	2.21	2.29	2.42	2.51	2.70	2.90	3.03
6,000	1.88	2.18	2.26	2.39	2.47	2.66	2.85	2.97
6,500	1.85	2.15	2.22	2.35	2.44	2.62	2.80	2.91
7,000	1.82	2.12	2.20	2.32	2.40	2.58	2.75	2.85
7,500	1.80	2.09	2.16	2.29	2.36	2.53	2.70	2.79
8,000	1.77	2.06	2.13	2.26	2.33	2.49	2.65	2.73
8,500	1.74	2.03	2.10	2.22	2.29	2.45	2.60	2.67
9,000	1.71	2.00	2.07	2.19	2.26	2.41	2.55	2.60[c]
9,500	1.68	1.97	2.04	2.15	2.22	2.37	2.49[c]	2.54
10,000	1.66	1.94	2.01	2.12	2.19	2.33	2.44	2.47
10,500	1.63	1.90	1.97	2.08	2.15	2.28	2.39	2.41
11,000	1.60	1.87	1.94	2.05	2.71	2.24[c]	2.33	
11,500	1.57	1.84	1.91	2.01	2.07	2.20	2.28	
12,000	1.55	1.81	1.87	1.98	2.04[c]	2.15		
12,500	1.52	1.78	1.84	1.94[c]	2.00	2.11		
13,000	1.49	1.75	1.81	1.91	1.96			
13,500	1.46	1.72[c]	1.78[c]	1.87	1.92			
14,000	1.43	1.68	1.74	1.84				
14,500	1.40	1.65	1.71					
15,000	1.37	1.62	1.68					
15,500	1.34[c]	1.59						
16,000	1.32							
16,500	1.29							
17,000	1.26							

[a]Temperature is held constant at the overall mean.

[b]Ration is the ratio of soilage to corn.

[c]Indicates a feeding period of more than 138 days. See Table 31.

Table 34. Estimated quantities of corn and soilage[a] that would be fed and the predict
intervals (temperature is held constant at the overall mean)

	30 days			60 days			90 days		
Ration[b]	Lbs. corn[c]	Lbs. soilage[d]	Lbs. gain[e]	Lbs. corn	Lbs. soilage	Lbs. gain	Lbs. corn	Lbs. soilage	L[illegible] g[illegible]
All soilage	--	2,549	40.0	--	5,232	100.8	--	8,048	13[illegible]
20:1	116	2,322	63.2	236	4,716	119.8	35[illegible]	7,150	16[illegible]
15:1	150	2,248	65.6	304	4,555	125.0	461	6,919	17[illegible]
10:1	212	2,115	70.1	427	4,270	134.2	646	6,463	1[illegible]
8:1	253	2,026	73.0	510	4,082	140.3	771	6,166	201
5:1	361	1,803	80.7	723	3,617	155.7	1,089	5,444	225
3:1	504	1,513	90.8	1,010	3,031	175.9	1,518	4,554	256
2:1	632	1,264	99.7	1,269	2,538	193.8	1,912	3,824	2[illegible]

[a]For each of the feed combinations there would also be fed a certain amount of the
of a pound per day.

[b]Ration is the ratio of soilage to corn.

[c]Derived from equation 41.

[d]The all soilage value was derived from equation 35, all other values were derived

[e]Derived from equation 19.

e predicted beef gains for eight selected stilbestrol rations for six different feeding

0 days		120 days			130 days			140 days		
Lbs. silage	Lbs. gain	Lbs. corn	Lbs. soilage	Lbs. gain	Lbs. corn	Lbs. soilage	Lbs. gain	Lbs. corn	Lbs. soilage	Lbs. gain
,046	136.1	--	10,997	164.5	--	12,010	170.6	--	13,037	175.2
,180	169.1	486	9,713	210.1	529	10,572	221.9	572	11,439	232.6
,919	177.3	623	9,338	222.0	677	10,158	235.1	732	10,983	247.3
,463	1[illegible]1.[illegible]	869	8,692	242.7	944	9,443	253.0	1,020	10,198	272.5
,166	201.4	1,035	8,276	256.0	1,123	8,986	272.7	1,212	9,698	288.7
,444	225.0	1,456	7,281	288.4	1,579	7,896	308.2	1,703	8,513	327.3
,554	256.4	2,026	6,053	320.2	2,198	6,594	352.5	2,369	7,106	375.2
,824	282.2	2,561	5,122	364.8	2,779	5,558	391.1	2,997	5,995	416.7

unt of the supplement shown in Table 3. This supplement would be fed at the rate of .2

re derived from equation 26.

Table 35. Estimated quantities of corn and soilage[a] that would be fed and the predicted feeding intervals (temperature is held constant at the overall mean)

Ration[b]	30 days			60 days			90 days		
	Lbs. corn[c]	Lbs. soilage[d]	Lbs. gain[e]	Lbs. corn	Lbs. soilage	Lbs. gain	Lbs. corn	Lbs. soilage	Lbs. gai
All soilage	--	3,125	60.9	--	6,326	111.6	--	[illegible],605	151.
20:1	134	2,689	71.6	272	5,441	132.9	413	8,257	153.
15:1	171	2,563	74.3	346	5,187	138.4	525	7,873	191.
10:1	234	2,344	79.1	475	4,746	147.8	721	7,207	205.
8:1	275	2,203	82.1	558	4,463	153.8	847	6,779	214.
5:1	374	1,868	89.3	758	3,788	167.9	1,153	5,763	234.
3:1	490	1,471	97.8	997	2,991	184.3	1,520	4,561	255.
2:1	581	1,163	104.3	119	2,370	196.8	1,812	3,625	275.

[a]For each of the feed combinations there would also be fed a certain amount of the s of a pound per day.

[b]Ration is the ratio of soilage to corn.

[c]Derived from equation 42.

[d]The all soilage value was derived from equation 36, all other values were derived

[e]Derived from equation 20.

the predicted beef gains for eight selected non-stilbestrol rations for six different
ean)

90 days		120 days			130 days			140 days		
Lbs. soilage	Lbs. gain	Lbs. corn	Lbs. soilage	Lbs. gain	Lbs. corn	Lbs. soilage	Lbs. gain	Lbs. corn	Lbs. soilage	Lbs. gain
[illegible],605	151.2	--	12,960	176.9	--	14,096	185.3	--	15,240	190.3
8,257	133.2	557	11,139	221.5	606	12,112	231.5	655	13,094	240.0
7,873	191.3	708	10,621	232.3	770	11,550	243.1	832	12,487	252.5
7,207	205.3	972	9,725	250.6	1,058	10,579	252.8	1,144	11,438	273.5
6,779	214.1	1,144	[illegible],153	262.0	1,245	9,957	275.1	1,346	10,768	286.6
5,763	234.6	1,558	7,792	2[illegible].3	1,696	8,481	303.1	1,835	9,176	316.3
4,561	255.1	2,061	6,184	317.7	2,240	6,737	334.2	2,432	7,295	348.9
3,625	275.7	2,464	4,929	339.3	2,687	5,347	356.8	2,914	5,827	372.2

mount of the supplement shown in Table 3. This supplement would be fed at the rate of .2

were derived from equation 26.

cussed above:

II. Without stilbestrol

$$(42)\quad C = 5{,}640.651186 + 1{,}630.0\alpha + 1.08874220T \pm 1{,}630.0\left[(-3{,}4605222 - \alpha - .00066794T)^2 - .03887249T - .00001616T^2 - .00679832H\right]^{1/2}.$$

Tables 36 and 37 are presented to show the estimated feeding periods for the various possible feed combinations presented earlier in Tables 6 and 7, respectively. These estimated feeding periods in Tables 36 and 37 have been used in Tables 6 and 7, respectively, as a basis for designating the relevant marginal rates of substitution.

Table 36. Predicted feeding time for various possible feeding combinations[a] for various levels of gain - with stilbestrol (temperature held constant at the overall mean)

	100 lbs. gain		200 lbs. gain		300 lbs. gain		350 lbs. gain		400 lbs. gain	
Lbs. corn	Lbs. soilage[b]	No. of days[c]	Lbs. soilage	No. of days	Lbs. soilage	No. of days	Lbs. soilage	No. of days	Lbs. soilage	No. of days
0	5,182	59.5								
100	4,456	54.2								
200	3,780	49.2	13,506	150.7[d]						
300	3,147	44.6	11,127	130.3						
400	2,550	40.2	9,652	118.2						
500	1,984	35.9	8,490	108.9						
600	1,447	31.8	7,503	101.1						
700	933[e]	27.9	6,632	94.3						
800			5,846	88.2						
900			5,126	82.6						
1,000			4,459	77.4						
1,100			3,836	72.5	13,732	180.0[d]				
1,200			3,250	67.9	11,382	158.9[d]				
1,300			2,696	63.4	9,954	146.6[d]				
1,400			2,170[e]	59.1	8,832	137.2[d]				
1,500					7,882	129.3				
1,600					7,045	122.3	13,192	189.0[d]		
1,700					6,291	116.1	11,201	170.9[d]		
1,800					5,601	110.3	9,879	159.3[d]		
1,900					4,962	104.9	8,822	150.1[d]		
2,000					4,366	99.8	7,918	142.3[d]		
2,100					3,806[e]	95.0	7,118	135.5	13,431	205.6[d]
2,200							6,396	129.2	11,412	186.7[d]
2,300							5,733	123.4	10,103	174.8[d]
2,400							5,119	118.0	9,063	165.4[d]
2,500							4,545	112.9	8,176	157.5[d]
2,600									7,393	150.5[d]
2,700									6,687	144.1[d]
2,800									6,040	138.2[d]
2,900									5,441[e]	132.7

[a]For each of the feed combinations there would also be fed a certain amount of the supplement shown in Table 3 fed at the rate of .2 of a pound per day.

[illegible]	[illegible]	[illegible]	[illegible]	108.9						
600	1,447	31.8	7,503	101.1						
700	933[e]	27.9	6,632	94.3						
800			5,846	88.2						
900			5,126	82.6						
1,000			4,459	77.4						
1,100			3,836	72.5	13,732	180.0[d]				
1,200			3,250	67.9	11,382	158.9[d]				
1,300			2,696	63.4	9,954	146.6[d]				
1,400			2,170[e]	59.1	8,832	137.2[d]				
1,500					7,882	129.3				
1,600					7,045	122.3	13,192	189.0[d]		
1,700					6,291	116.1	11,201	170.9[d]		
1,800					5,601	110.3	9,879	159.3[d]		
1,900					4,962	104.9	8,822	150.1[d]		
2,000					4,366	99.8	7,918	142.3[d]		
2,100					3,806[e]	95.0	7,118	135.5	13,431	205.6[d]
2,200							6,396	129.2	11,412	186.7[d]
2,300							5,733	123.4	10,103	174.8[d]
2,400							5,119	118.0	9,063	165.4[d]
2,500							4,545	112.9	8,176	157.5[d]
2,600									7,393	150.5[d]
2,700									6,687	144.1[d]
2,800									6,040	138.2[d]
2,900									5,441[e]	132.7

[a]For each of the feed combinations there would also be fed a certain amount of the supplement shown in Table 3 fed at the rate of .2 of a pound per day.

[b]Derived from equation 21.

[c]Derived from equation 39.

[d]The estimated feeding period exceeds the 136 day average feeding period in the experiment.

[e]The feed combination at this point exceeds the 2:1 ration and, hence, lies outside the limits of the experiment.

Table 37. Predicted feeding time for various possible feeding combinations[a] for various levels of gain - without stilbestrol (temperature held constant at the overall mean)

Lbs. corn	100 lbs. gain Lbs. soilage[b]	100 lbs. gain No. of days[c]	200 lbs. gain Lbs. soilage	200 lbs. gain No. of days	300 lbs. gain Lbs. soilage	300 lbs. gain No. of days	350 lbs. gain Lbs. soilage	350 lbs. gain No. of days
0	5,526	52.6	17,773	161.9[d]				
100	4,677	47.9	15,441	144.8[d]				
200	3,854	43.4	13,236	128.6				
300	3,056	39.2	11,623	117.5				
400	2,280	35.1	10,262	108.5				
500	1,524	31.1	9,051	100.7				
600	786[e]	27.3	7,943	93.8				
700			6,912	87.5				
800			5,941	81.7				
900			5,020	76.4				
1,000			4,140	71.3				
1,100			3,296	66.6				
1,200			2,482	62.1				
1,300			1,696[e]	57.8				
1,400								
1,500					11,488	151.9[d]		
1,600					9,350	136.1		
1,700					7,847	125.6		
1,800					6,589	117.3		
1,900					5,474	110.1		
2,000					4,456	103.8		
2,100					3,510[e]	98.1		
2,200								
2,300								
2,400							8,266	148.9[d]
2,500							6,492	136.0
2,600							5,161[e]	126.9

[a]For each of the feed combinations there would also be fed a certain amount

800	5,941	81.7				
900	5,020	76.4				
1,000	4,140	71.3				
1,100	3,296	66.6				
1,200	2,482	62.1				
1,300	1,696[e]	57.8				
1,400						
1,500			11,488	151.9[d]		
1,600			9,350	136.1		
1,700			7,847	125.6		
1,800			6,589	117.3		
1,900			5,474	110.1		
2,000			4,456	103.8		
2,100			3,510[e]	98.1		
2,200						
2,300						
2,400					8,266	148.9[d]
2,500					6,492	136.0
2,600					5,161[e]	126.9

[a]For each of the feed combinations there would also be fed a certain amount of the supplement shown in Table 3 fed at the rate of .2 of a pound per day.

[b]Derived from equation 22.

[c]Derived from equation 40.

[d]The estimated feeding period exceeds the 138 day average feeding period of the experiment.

[e]The feed combination at this point exceeds the 2:1 ration and, hence, lies outside the limits of the experiment.

GRADE FUNCTION

In the previous sections, the analysis has dealt with how feed resources (corn and soilage) may be combined to produce various levels of beef gain and the expected beef gains for feeding periods of various length. Very little consideration was given to the "quality" (grade) of the beef steers when fed different soilage-corn rations for various length feeding periods. The central point in this section will be to estimate beef grades for beef steers fed different soilage-corn rations. Once a functional relationship has been determined between beef grades and feed inputs it will be possible to construct iso-grade contours and to derive the marginal rates of substitution of corn for soilage in producing a given grade of beef.

The procedure that was adopted was to estimate the functional relationship, Grade = g(pounds of corn fed, pounds of soilage fed). However, in order to estimate this functional relationship it was necessary to code the beef grades which were measured in the usual subjective terms such as high standard, average good, low choice, etc. The various beef steer grades were coded by using a 10 year average yearly market price of the various slaughter steer grades. Specifically, the yearly average prices for the various grades of slaughter steers at the Chicago market for the 10 year period, 1951-1960, were listed and then the 10 year average price for each of the separate grades was determined (56). The 10 year average yearly price for each beef steer grade was considered to be the value of the <u>average</u> grade of that particular grade. For example, if the 10 year average yearly price for good steers at the Chicago market was $20.00, then this price was considered to be the value for <u>average</u> good steers. The <u>high</u> and <u>low</u> grade values for each grade were then determined by making a linear interpolation between the <u>average</u> grade values for the different grades. The grade index that was used to code the subjective grade terms is shown in Table 38. The computed range on each of the various beef grades is shown in Table 39.

After the observed subjective grade terms had been coded with numerical values, then for each lot of steers, the grade value of the steers at the beginning of the feeding experiment was subtracted from each of the observed grade values. This procedure gave a grade series in terms of the change in beef grade since the beginning of the feeding period. A quadratic function was then used to express the functional relationship between the change in beef grade (Q') and the consumption of various quantities of the two feeds -- corn (C) and soilage (F). This relationship was estimated for both the stilbestrol and non-stilbestrol rations. The overall*

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*Since only two grade observations were made in 1957, the estimated grade functions are based only on the combined feeding periods of 1958 and 1959. Hence, "overall" refers to the combined feeding periods of 1958 and 1959 at any one location.

Table 38. An index for coding market grades of slaughter steers[a]

Slaughter steer grades		Numerical code
Prime:	High	29.53
	Average	28.87
	Low	28.21
Choice:	High	27.55
	Average	26.87
	Low	26.13
Good:	High	25.38
	Average	24.64
	Low	23.70
Standard:	High	22.75
	Average	21.81
	Low	20.93
Utility:	High	20.04
	Average	19.16
	Low	18.28

[a]The numerical coding value for the average grade of each particular slaughter grade is the 10 year, 1951-1960, average yearly price for that grade of slaughter steers at Chicago (56). The coding values for the high and low grades of each particular slaughter grade are obtained by making a linear interpolation between the average values of each of the particular slaughter grades.

equations for estimating the change in grade of beef steers since the beginning of the feeding period for the stilbestrol and non-stilbestrol rations are:

I. With stilbestrol

(43) $Q' = .0024655079C + .0000220680F - .0000003510C^2$

II. Without stilbestrol

(44) $Q' = .0016294178C + .0000270836F - .0000000330C^2$.

Table 39. The range on the index values for market grades of slaughter steers[a]

Slaughter steer grades	Range of coded values for subjective slaughter steer grades
	$29.20 \le$ High Prime
Prime:	$28.54 \le$ Average Prime < 29.20
	$27.87 \le$ Low Prime < 28.54
	$27.21 \le$ High Choice < 27.87
Choice:	$26.50 \le$ Average Choice < 27.21
	$25.75 \le$ Low Choice < 26.50
	$25.01 \le$ High Good < 25.75
Good:	$24.17 \le$ Average Good < 25.01
	$23.22 \le$ Low Good < 24.17
	$22.28 \le$ High Standard < 23.22
Standard:	$21.37 \le$ Average Standard < 22.28
	$20.48 \le$ Low Standard < 21.37
	$19.60 \le$ High Utility < 20.48
Utility:	$18.72 \le$ Average Utility < 19.60
	Low Utility < 18.72

[a]The range of coded values for each subjective slaughter steer grade was obtained by making a linear interpolation between each of the grade values in Table 38.

In the above equations, C is the total intake of corn in pounds measured from the beginning of the feeding period to each particular observation period when an observation was made on grade. The feeder steers were first graded* at the

*In 1958, the feeder steers were graded at the beginning of the feeding experiment on both a feeder and slaughter steer basis. However, in 1959 the feeder steers were graded at the beginning of the feeding experiment on only a feeder basis. In order to construct a (continued on next page)

beginning of the feeding period. The next two grade observations were made at six week intervals, after which the beef steers were graded every 21 days until the end of the feeding experiment. F is total pounds of soilage intake measured in the same manner as was corn. Q' is the change in grade of the beef steer since the beginning of the feeding experiment.

The coefficient of determination, standard errors and the "t" values for the overall stilbestrol and non-stilbestrol grade functions are presented in Tables 40 and 41, respectively. The coefficient of determination is .9066 for the stilbestrol function and .8420 for the non-stilbestrol function. Certain of the variables in both the stilbestrol and non-stilbestrol functions are acceptable only at a very low level of probability, nevertheless, these variables have been retained in the function since they appear to be consistent with nutrition and production logic.

If a constant term is added to the change in grade functions (i.e., equations 43 and 44) and if this constant term represents the grade of the beef steers at the beginning of

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(Continued from previous page) grade surface it is necessary that the beef grades all be on the same basis. Therefore, it was necessary to convert the first grade observations in 1959 from a feeder basis to a slaughter basis. The 1958 data where the feeder steers were graded at the beginning of the feeding experiment on both a feeder and slaughter steer basis was used as a basis for converting the first grade observations in 1959 from a feeder to a slaughter basis. Thereafter, only the grade observations that were on a slaughter basis were used to determine the beef grade surface.

Table 40. Coefficient of determination, standard errors and "t" values for the overall stilbestrol grade function (equation 43)

R^2	Independent variable	Standard error of regression coefficient	"t" value	Level of significance
.9066	C	.0002031230	12.138	$p < .001$
	F	.0000128474	1.718	$.05 < p < .10$
	C^2	.0000000928	3.782	$p < .001$

Table 41. Coefficient of determination, standard errors and "t" values for the overall non-stilbestrol grade function (equation 44)

R^2	Independent variable	Standard error of regression coefficient	"t" value	Level of significance
.8420	C	.0002638487	6.176	$p < .001$
	F	.0000152410	1.777	$.05 < p < .10$
	C^2	.0000000330	.266	$p > .50$

the feeding period, then the equations with the constant term added can be used to predict the slaughter grade of good-to-choice feeder steers after being fed various quantities of corn and soilage. The predicted grade values can then be interpreted in subjective grade terms with the aid of Table

39.

The overall average grade value of the beef steers at the beginning of the feeding experiment was 21.67. When this value of 21.67 is used as the constant term in equations 43 and 44, then the overall grade functions (Q) for the stilbestrol and the non-stilbestrol rations can be written as:

I. With stilbestrol

$$(45)\quad Q = 21.67 + .0024655079C + .0000226068OF - .00000035100C^2$$

II. Without stilbestrol

$$(46)\quad Q = 21.67 + .0016294178O + .0000270836F - .0000000330C^2$$

where Q is the predicted slaughter grade which can be interpreted in subjective grade terms with the use of Table 39.

Iso-grade Contours

The beef iso-grade equations can be derived for the stilbestrol and non-stilbestrol rations from the two overall grade equations 45 and 46, respectively. The beef iso-grade equations are:

I. With stilbestrol

$$(47)\quad F = \frac{Q - 21.67 - .0024655079C + .00000035100C^2}{.0000220680}$$

II. Without stilbestrol

$$(48)\quad F = \frac{Q - 21.67 - .0016294178C + .0000000330C^2}{.0000270836}$$

The iso-grade equations can be used to determine the iso-grade contours that specify the various quantities of corn and soilage required to attain a given grade of beef. The slope of the iso-grade contours is the substitution rate between the two feeds in the production of a given grade of beef. The equations for predicting the marginal rate of substitution of corn for soilage in the production of a given grade of beef can be obtained from the iso-grade equations by taking the partial derivative of soilage with respect to corn. The equations for predicting the marginal rates of substitution of corn for soilage in the production of a given grade of beef are:

I. With stilbestrol

$$(49) \quad \frac{\partial F}{\partial C} = \frac{.0024655079 - .0000007020C}{.0000220680}$$

II. Without stilbestrol

$$(50) \quad \frac{\partial F}{\partial C} = \frac{.0016294178 - .0000006600C}{.0000270836}.$$

Beef iso-grade schedules and the marginal rates of substitution associated with them have been derived for the following beef grades: high standard, low good, average good, high good and low choice. The beef iso-grade schedules and associated marginal rates of substitution are presented in Tables 42 and 43 for the overall stilbestrol and non-stilbestrol functions, respectively. The iso-grade schedules have been plotted in Figure 24 for the overall stilbestrol function

Table 42. Iso-grade schedules, derived from the overall stilbestrol grade functi
combinations[a] and marginal rates of substitution of corn for soilage a
for 850 pound good-to-choice feeder steers (temperature is held consta

	High standard[b]			Low Good[b]			Average Good[b]	
Lbs. corn	Lbs. soilage	Ration[d]	$\frac{\partial F}{\partial C}$[c]	Lbs. soilage	Ration	$\frac{\partial F}{\partial C}$	Lbs. soilage	Ration
0								
100								
200	27,231[c]	136.16	105.36					
300	16,854[e]	56.18	102.18					
400	6,795	14.49	100.00					
500								
600				30,680[c]	51.13	92.63		
700				21,575[e]	30.82	89.46		
800				12,789	15.99	86.2[illegible]		
900				4,320	4.80	83.10		
1,000								
1,100								
1,200							23,418[e]	19.52
1,300							16,222[e]	12.48
1,400							9,344	6.67
1,500							2,784	1.86[f]
1,600								
1,700								
1,800								
1,900								
2,000								
2,100								
2,200								
2,300								
2,400								
2,500								
2,600								
2,700								
2,800								
2,900								
3,000								

[a]For each of the feed combinations there would also be fed a certain amount
be fed at the rate of .2 of a pound per day.

[b]The numerical value of the subjective slaughter grades used in deriving the
particular grade as shown in Table 38.

[c]Indicates the marginal rate of substitution of corn for soilage.

[d]Ration is the ratio of soilage to corn.

[e]Estimated time required to consume this combination of corn and soilage exc

[f]All feed combinations at this point exceed the 2:1 ration and, hence, are o

e function (equation 45), showing possible feed
cilage at five slaughter steer grade levels,
d constant at the overall mean)

...ge Good[b]		High Good[b]			Low Choice[b]		
...ation	$\frac{\partial F}{\partial C}$	Lbs. scilage	Ration	$\frac{\partial F}{\partial C}$	Lbs. scilage	Ration	$\frac{\partial F}{\partial C}$
17.52	73.55						
12.48	70.37						
6.67	67.19						
1.86[f]	64.01						
		18,535[e]	10.30	54.47			
		13,257[e]	6.98	51.29			
		8,287	4.14	48.11			
		3,636	1.73[f]	44.93			
					19,135[e]	7.36	29.02
					16,392[e]	6.07	25.84
					13,967[e]	4.99	22.66
					11,860[e]	4.09	19.47
					10,071[e]	3.36	16.30

amount of the supplement shown in Table 3. This supplement would

ving the iso-grade schedules is the average value of each

lage exceeds the 136 day average feeding period.

e, are outside the limits of the experiment.

Table 43. Iso-grade schedules, derived from the overall non-stilbestrol grade function (
combinations[a] and marginal rates of substitution of corn for soilage at five s
850 pound good-to-choice feeder steers (temperature is held constant at the ov

	High Standard[b]			Low Good[b]			Average Good[b]		
Lbs. corn	Lbs. soilage	Ration[d]	$\frac{\partial F}{\partial C}$[c]	Lbs. soilage	Ration	$\frac{\partial F}{\partial C}$	Lbs. soilage	Ration	$\frac{\partial F}{\partial C}$
0									
100									
200									
300	21,938[e]	73.13	59.43						
400	16,007[e]	40.02	59.19						
500	10,100	20.20	58.94						
600	4,218	7.03	58.45						
700									
800									
900				21,795[e]	24.22	57.97			
1,000				16,010[e]	16.01	57.72			
1,100				10,250	9.31	57.48			
1,200				4,515	3.76	57.23			
1,300									
1,400									
1,500							22,161[e]	14.77	56.50
1,600							16,523[e]	10.33	56.26
1,700							10,909[e]	6.42	56.02
1,800							5,320	2.96	55.77
1,900									
2,000									
2,100									
2,200									
2,300									
2,400									
2,500									
2,600									
2,700									
2,800									
2,900									
3,000									

[a]For each of the feed combinations there would also be fed a certain amount of the su
would be fed at the rate of .2 of a pound per day.

[b]The numerical value of the subjective slaughter grades used in deriving the iso-grac
particular grade shown in Table 38.

[c]Indicates the marginal rate of substitution of corn for soilage.

[d]Ration is the ratio of soilage to corn.

[e]The estimated time required to consume this combination of corn and soilage exceeds

[f]All feed combinations at this point exceed the 2:1 ration and, hence, are outside th

ilbestrol grade function (equation 46), showing possible feed
corn for scilage at five slaughter steer grade levels, for
is held constant at the overall mean)

Average Good[b]			High Good[b]			Low Choice[b]		
Lbs. scilage	Ration	$\frac{\partial F}{\partial C}$	Lbs. scilage	Ration	$\frac{\partial F}{\partial C}$	Lbs. scilage	Ration	$\frac{\partial F}{\partial C}$
2,161[e]	14.77	56.50	—					
6,523[e]	10.33	56.26						
0,909[e]	6.42	56.02						
5,320	2.96	55.77						
			21,537[e]	10.77	55.28			
			16,021[e]	7.63	55.04			
			10,529[e]	4.79	54.80			
			5,062	2.20	54.55			
						21,893[e]	8.76	54.06
						16,498[e]	6.35	53.82
						11,129[e]	4.12	53.58
						5,783	2.07	53.33
						462	.16[f]	53.09

a certain amount of the supplement shown in Table 3. This supplement

d in deriving the iso-grade schedules is the <u>average</u> value of each

ilage.

corn and scilage exceeds the 138 day average feeding period.

and, hence, are outside the limits of the experiment.

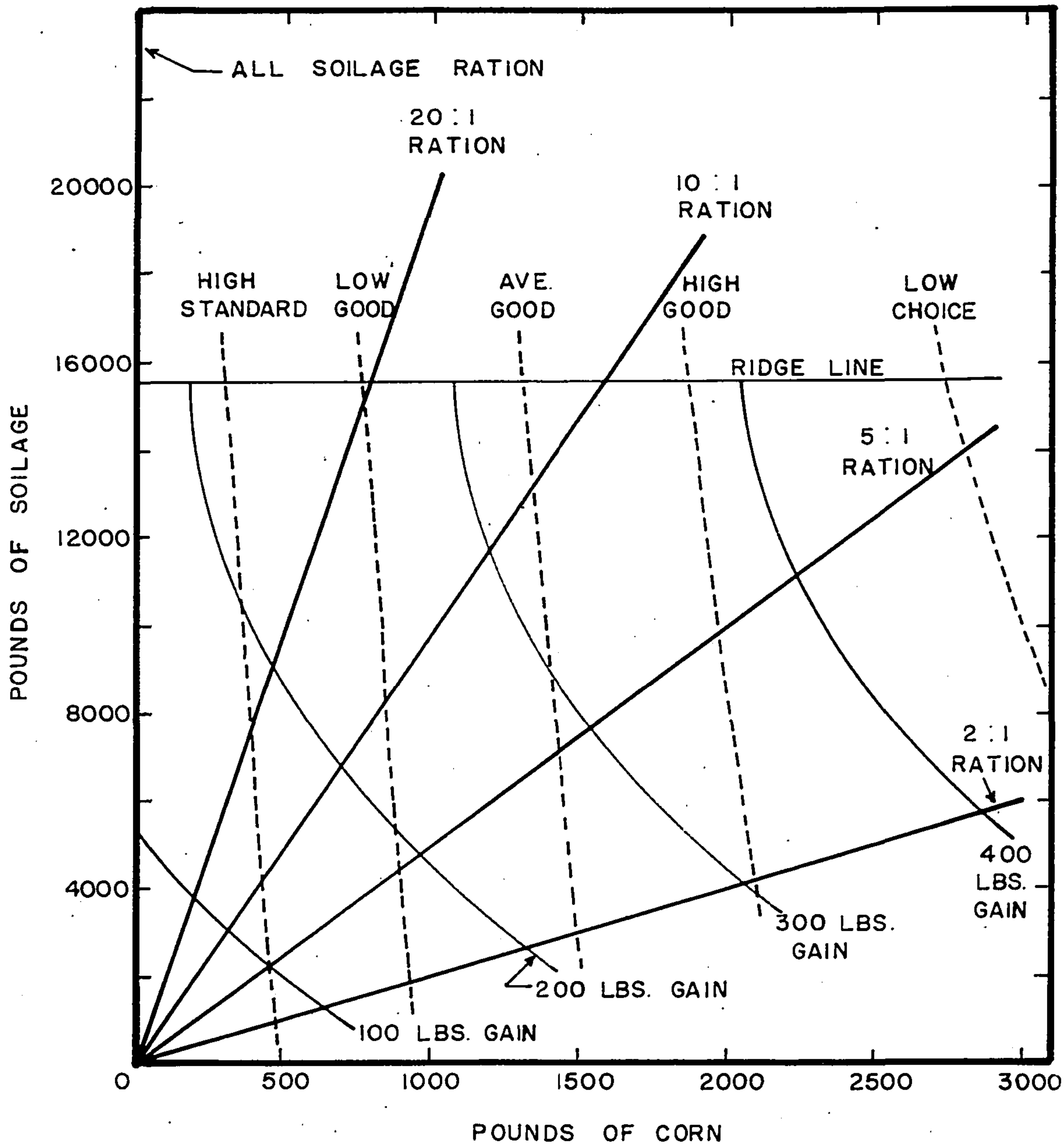

Figure 24. Gain isoquants and iso-grade contours for the stilbestrol rations (temperature held constant at the overall mean)

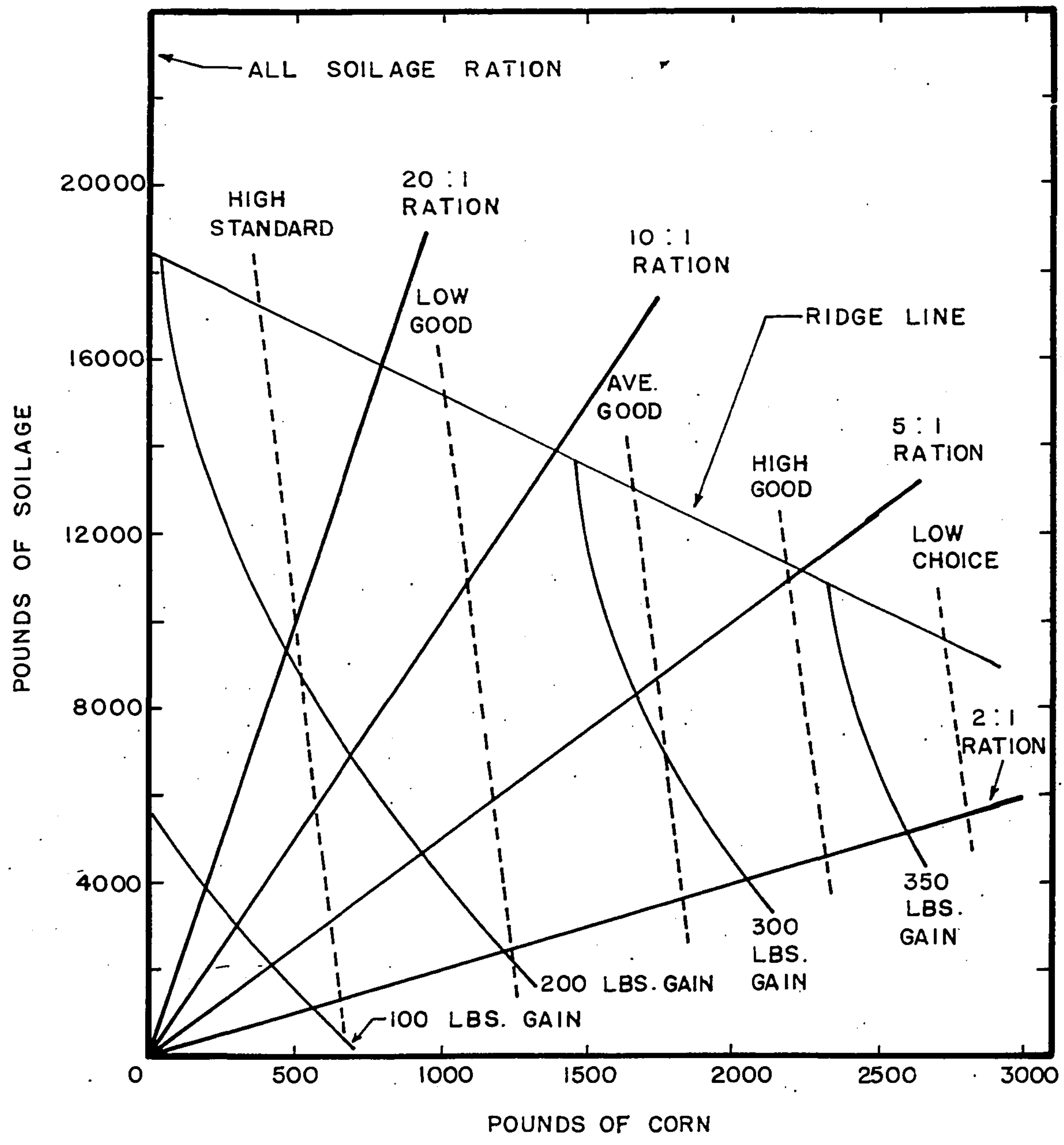

Figure 25. Gain isoquants and iso-grade contours for the non-stilbestrol rations (temperature held constant at the overall mean)

and in Figure 25 for the overall non-stilbestrol function. As mentioned earlier the slope of the iso-grade curves at any given point indicates the rate at which corn substitutes for soilage in the production of a given grade of beef. The curvature of the iso-grade curves, as indicated in both Figures 24 and 25, changes very little, suggesting that the substitution rates between the two feeds in the production of a given grade of beef is nearly constant. The iso-grade curves in both Figures 24 and 25 are slightly convex to the origin indicating that the marginal rates of substitution of corn for soilage for any given grade of beef are at a diminishing rate.

By superimposing the gain isoquants over the iso-grade curves, it is possible to get some idea of the relationship between the levels of beef gains and beef grades. Figures 24 and 25 show the predicted gain isoquants superimposed over the predicted iso-grade curves for the stilbestrol and non-stilbestrol rations, respectively. In Figure 24, the average good iso-grade contour is represented by a coded numerical grade value of 24.64 as shown in Table 38. However, in subjective grade terms the average good grade, as well as all other grades, can be thought of as extending over a range of numerical values. The average good grade in coded numerical values, as shown in Table 39, extends from 24.17 to 25.01. Furthermore, the entire grade surface can be broken down into grade "areas" as indicated in Table 39. The average good grade in Figure 24, for example, extends both above and below the average good iso-grade contour. Therefore, each of the iso-grade curves can be thought of as a "wide band" extending over the grade surface denoting the various subjective beef grade "areas" such as high standard, low good, average good, etc.

PROFIT MAXIMIZATION

In the last section an attempt has been made to estimate the beef grade surface and iso-grade contours. The procedure there was to merely estimate the grade of slaughter steers when fed different soilage-corn rations. No attempt was made, however, to estimate the value of the slaughter steers at the end of a given feeding period. The purpose of this section is 1) to estimate the expected profits from feeding various soilage-corn rations for various feeding periods with different soilage-corn price conditions, 2) to estimate the optimum feeding period for different soilage-corn rations with different soilage-corn price conditions and 3) to estimate the optimum soilage-corn ration that will maximize profits for different soilage-corn price conditions.

The price for which beef cattle will sell at the end of a given feeding period depends, <u>certeris paribus</u>, upon their grade. The higher the beef cattle grade the higher will be the selling price (see Figure 26). Thus one of the main objects of fattening beef cattle is to improve their grade (quality). While the price of beef cattle will vary between grades, the price of the different grades will also vary over any given feeding period because of seasonal price changes. Therefore, the value or the price for which the beef cattle will sell at any given time depends upon the grade of the cattle and the price for that particular grade.

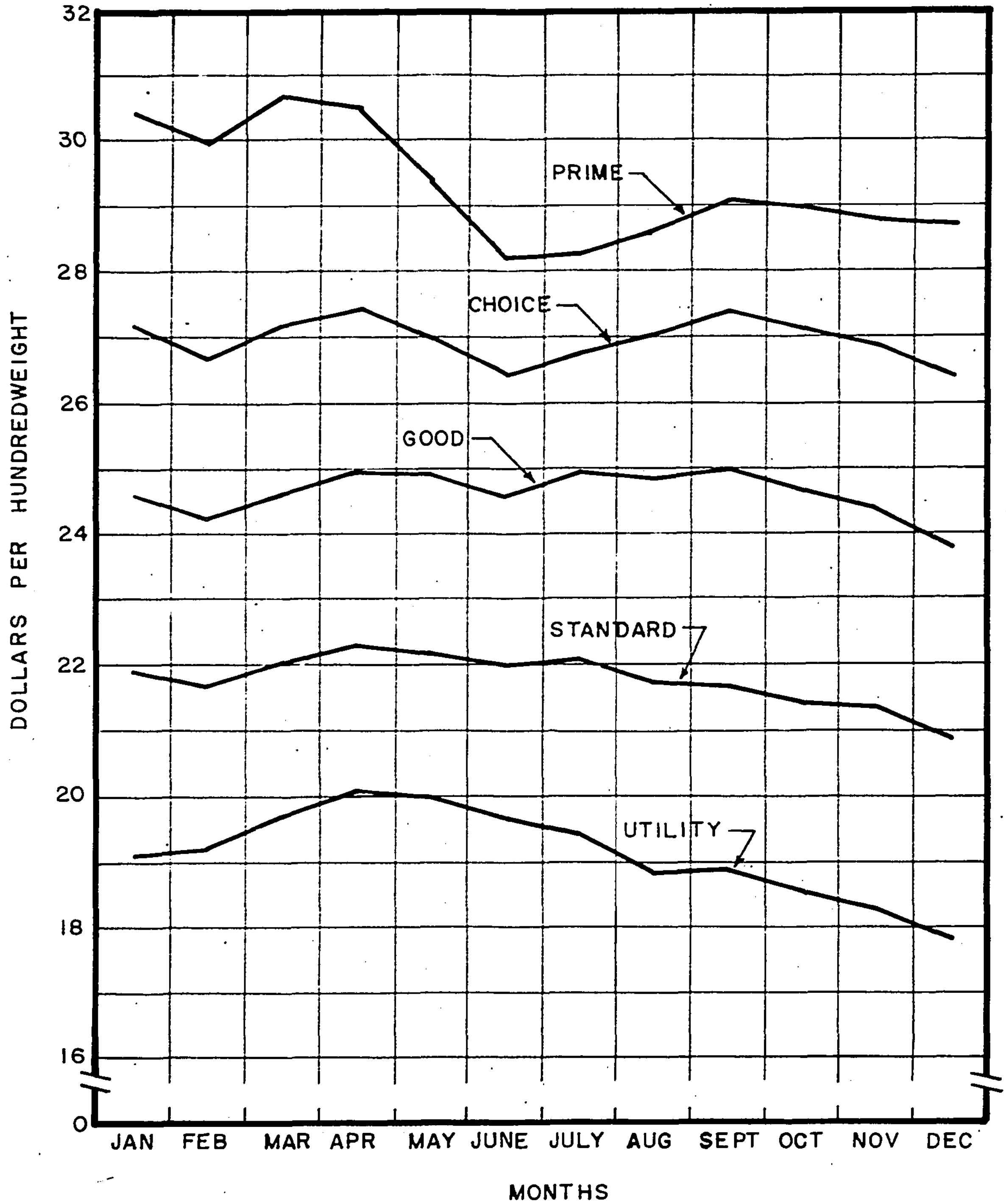

Figure 26. Seasonal change in slaughter steer prices at Chicago, 1951-1960 average

In order to estimate the price for which slaughter steers will sell at any given time, the functional relationship that expresses the price of slaughter steers as a function of the quantity of soilage consumed and time was computed. However, to estimate this functional relationship it was necessary to have a price series that represents the grade of the beef steers during the feeding period as well as the market price associated with the grade. Since the beef steers were graded at definite intervals throughout the beef feeding experiment, these subjective grade terms can be replaced with the market price for that grade at the time the steers were appraised. This procedure will allow the subjective grade terms to be given a numerical value for analysis purposes and also furnishes a price series that represents the value or price of the beef steer at various stages of the feeding period.

The price of steers for this analysis is based on weekly Chicago prices (56). For each week throughout the beef feeding experiment, a 10 year (1951-1960) weekly average price was computed for each of the various grades of slaughter and feeder steers. The 10 year average weekly price of each grade was considered to be the <u>average</u> price of that particular grade. For example, if for the second week in August the 10 year average weekly price for choice slaughter steers at Chicago was $25.00 per hundred pounds, then this price was considered to be the price for average choice slaughter steers. The price for the high and low grades of each particular grade were then determined by making a linear interpolation between the average grade values.

After the 10 year average weekly price had been computed for each of the beef grades, the subjective grade observations were then replaced with the 10 year average weekly price that corresponded with the week in which the grade observation was made. In some instances the beef steers were graded on both a feeder and slaughter basis while in most cases the steers were graded on either a feeder or a slaughter basis. In the case where the beef steers were graded on both a feeder and a slaughter basis, the basis that resulted in the highest price was the one that was used in the analysis. This procedure assumes that if a beef steer is to be sold he will be sold on the grade basis that will bring the greatest returns.

In order to estimate the change in the price of beef steers from the beginning of the feeding period, a quadratic function was used to determine the functional relationship, the change in the price of beef steers (P') = p'(pounds of soilage, time in days). This relationship was estimated for both the stilbestrol and the non-stilbestrol rations. The "overall"* change in price equations for the stilbestrol and

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*"Overall" refers to the combined feeding periods of 1958 and 1959 at any one location.

the non-stilbestrol rations are:

I. With stilbestrol

(51) $P' = -.0000040158F + .0000382807T + .0000017537T^2 - .0000000048FT$

II. Without stilbestrol

(52) $P' = -.0000004996F - .0002393978T + .0000036576T^2 - .0000000281FT.$

In the above equations, F refers to pounds of soilage, T refers to time in days and P' refers to the change in the price of beef steers measured in cents per pound. All of the above variables are measured from the beginning of the feeding period to each particular observation period when an observation was made on grade.**.

The price series (P') that was used in this analysis was obtained by subtracting the price of the steers at the beginning of the feeding period from all price values in the series. Thus the first price observation value would be zero. Consequently, the price equations 51 and 52 have been estimated without a constant term.

The coefficient of determination, standard errors and the "t" values for the stilbestrol and non-stilbestrol price functions are presented in Tables 44 and 45, respectively.

If a constant term is added to equations 51 and 52 and

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*See page 130ff. for additional information on the intervals of the grade observations.

Table 44. Coefficient of determination, standard errors and "t" values for the overall stilbestrol price function (equation 51)

R^2	Independent variable	Standard error of regression coefficient	"t" value	Level of significance
.8218[a]	F	.0000005145	7.806	$p < .001$
	T	.0000507697	.754	$.40 < p < .50$
	T^2	.0000003979	4.407	$p < .001$
	FT	.0000000039	1.228	$.20 < p < .40$

[a]The coefficient of determination is based on the raw sum of squares.

Table 45. Coefficient of determination, standard errors and "t" values for the overall non-stilbestrol price function (equation 52)

R^2	Independent variable	Standard error of regression coefficient	"t" value	Level of significance
.7484[a]	F	.0000025374	.197	$p > .50$
	T	.0000018953	1.263	$.20 < p < .40$
	T^2	.0000000168	2.180	$.025 < p < .05$
	FT	.0000000225	1.250	$.20 < p < .40$

[a]The coefficient of determination is based on the raw sum of squares.

if this constant term is the value of the beef steers at the beginning of the feeding period, then the price functions (*i.e.*, those functions with the constant term added) can be used to predict the price at which the beef steers will sell. The average price of the feeder steers at the beginning of the feeding period was 25 cents per pound. When this value of 25 cents is used as the constant term, the price function (P) for the stilbestrol and non-stilbestrol rations can be written as:

I. With stilbestrol

$$(53) \quad P = .2500 - .0000040158F + .0000382807T + .0000017537T^2 - .0000000048FT$$

II. Without stilbestrol

$$(54) \quad P = .2500 - .0000004996F - .0002393978T + .0000036576T^2 - .0000000281FT.$$

Similarly, if a constant term is added to the production functions in equations 19 and 20 and if this constant term represents the average weight of the steers at the beginning of the feeding period, then the production functions with this constant term can be used to predict the total weight (W) of the beef steers. The equations for estimating the total weight (W) of the beef steers for the stilbestrol and non-stilbestrol rations can be written as:

I. With stilbestrol

$$(55)\quad W = 850.00 + .11637150C + .02316051F - .0000049955C^2 - .0000007455F^2 + .000000374CF - 1.2236046H$$

II. Without stilbestrol

$$(56)\quad W = 850.00 + .14971812C + .02128774F - .0000122612C^2 - .0000005775F^2 - .0000037907CF - 2.2005042H.$$

Profit Function

Profit is defined as the difference between total revenue and the total expenditure for all inputs. The profit function as it is related to beef cattle can be depicted as:

$$(57)\qquad \pi = WP - P_C C - P_F F - .2TP_S - K$$

where π refers to the profit, W refers to the total weight of the steer, P refers to the selling price, P_C refers to the price of corn, C refers to the pounds of corn fed, P_F refers to the price of soilage, F refers to the pounds of soilage fed, T refers to time in days, P_S refers to the price of the supplement and K is the value of the feeder steer at the beginning of the feeding period.

Thus the overall profit functions for the stilbestrol and non-stilbestrol rations are:

I. With stilbestrol

$$(58)\quad \pi = (850.00 + .11637150C + .02316051F - .0000049955C^2 - .0000007455F^2 + .0000000374CF - 1.2236046H)\,(.2500 - .0000040158F + .0000382807T + .0000017537T^2 - .0000000048FT) - P_C C - P_F F - .2TP_S - K$$

II. Without stilbestrol

$$(59)\quad \pi = (850.00 + .14971812C + .02128774F - .000122612C^2 - .0000005775F^2 - .0000037907CF - 2.2005042H)\,(.2500 - .0000004996F - .0002393978T + .0000036576T^2 - .0000000281FT) - P_C C - P_F F - .2TP_S - K.$$

The profit equations can be used to estimate profits from feeding any given soilage-corn ration, from the all soilage ration to the 2:1 soilage-corn ration, for any given feeding period within the pasture growing season. The estimated profits from feeding, for example, the 10:1 soilage-corn stilbestrol ration for 140 days can be determined if the cost of the feeder steer and the prices of the feed inputs are known. The quantity of corn that will be fed in the 10:1 soilage-corn stilbestrol ration can be determined from equation 41. The soilage value corresponding to this corn value is then readily determined from the ration equation 26. Therefore, given the cost of the feeder steer and the prices of the feed inputs, the expected profits can then be predicted.

The profit equation can also be used to estimate profits from feeding some given total quantity of feed of a specific ration. The time required to consume this given total quantity of feed of a given ration can be determined from the time equations in Table 28 or Table 29. A time equation can be derived for rations other than those listed by following the same procedure as was used in deriving the particular equations listed in Tables 28 or 29. Again, if the cost of the feeder steer and the prices of the feed inputs are known, then the expected profits can be determined.

The expected profits from feeding various stilbestrol rations for 140, 130, 120 and 90 days with various feed price assumptions are presented in Tables 46, 47, 48 and 49, respectively. In Table 46, a feeder steer fed the 20:1 ration for 140 days is predicted to consume 11,439 pounds of soilage, 572 pounds of corn and 28 pounds of supplement. At the end of the 140 day feeding period, the steer is predicted 1) to weigh 1,083 pounds, 2) to grade low good, 3) to sell for a price of $23.62 per hundredweight and 4) to be worth $255.67. The steer at the beginning of the feeding period has been valued at $25.00 per hundredweight for a total value of $212.50. If the price of soilage is $2.00 per ton and the price of corn is $1.00 per bushel, then the total feed costs for feeding a steer 140 days will be $22.63, which includes

Table 46. Predicted total feed consumption, total weight, grade, selling price, total revenue, total feed costs, and net revenue for good-to-choice feeder steers, weighing 850 pounds at the outset, for eight selected soilage-corn stilbestrol rations fed for 140 days (equation 58)[a]

Feed consumption[b] (lbs.)		Ration: All soilage		20:1		15:1		10:1	
Soilage		13,037		11,439		10,983		10,198	
Corn		0		572		732		1,020	
Supplement[c]		28		28		28		28	
Cost of feeder steer[d]		$212.50		$212.50		$212.50		$212.50	
Final weight		1,025		1,083		1,097		1,123	
Grade[e]		Av. standard		High standard		Low good		Low good	
Selling price		$22.87		$23.62		$23.83		$24.20	
Total revenue		$234.46		$255.67		$261.49		$271.63	
Price of soilage ($/ton)	Price of corn ($/bu.)	Total feed cost ($)	Net revenue ($)	Total feed cost ($)	Net revenue ($)	Total feed cost ($)	Net revenue ($)	Total feed cost[f] ($)	Net revenue ($)
1.00	--	7.50	14.46						
	.75			14.36	28.81	16.28	32.71	19.74	39.40
	1.00			16.91	26.26	19.55	29.45	24.29	34.84
	1.25			19.47	23.71	22.81	26.18	28.84	30.29
	1.50			22.02	21.15	26.08	22.91	33.40	25.74
	1.75			24.57	18.60	29.35	19.64	37.95	21.19
2.00	--	14.02	7.95						
	.75			20.08	23.09	21.77	27.22	24.84	34.30
	1.00			22.63	20.54	25.04	23.95	29.39	29.74
	1.25			25.19	17.99	28.31	20.68	33.94	25.19
	1.50			27.74	15.43	31.57	17.42	38.50	20.64
	1.75			30.29	12.88	34.84	14.15	43.05	16.09
3.00	--	20.54	1.43						
	.75			25.80	17.37	27.26	21.73	29.94	29.20
	1.00			28.35	14.82	30.53	18.46	34.49	24.65
	1.25			30.90	12.27	33.80	15.19	39.04	20.09
	1.50			33.46	9.71	37.07	11.92	43.59	15.54
	1.75			36.01	7.16	40.33	8.66	48.15	10.99
4.00	--	27.05	-5.09						
	.75			31.52	11.65	32.75	16.24	35.04	24.10
	1.00			34.07	9.10	36.02	12.97	39.59	19.55
	1.25			36.62	6.55	39.29	9.70	44.14	14.99
	1.50			39.18	3.99	42.56	6.43	48.69	10.44

	.75			31.52	11.65	32.75	16.24	35.04	24.10
	1.00			34.07	9.10	36.02	12.97	39.59	19.55
	1.25			36.62	6.55	39.29	9.70	44.14	14.99
	1.50			39.18	3.99	42.56	6.43	48.69	10.44
	1.75			41.73	1.44	45.83	3.16	53.25	5.89
5.00	--	33.57	-11.61						
	.75			37.24	5.93	38.24	10.75	40.13	19.00
	1.00			37.79	3.30	41.51	7.48	44.69	14.45
	1.25			42.34	.83	44.78	4.21	49.24	9.89
	1.50			44.90	-1.72	48.05	.94	53.79	5.34
	1.75			47.45	-4.28	51.32	-2.33	58.35	.79
6.00	--	40.09	-18.13						
	.75			42.96	.22	43.73	5.26	45.23	13.90
	1.00			45.51	-2.34	47.00	1.99	49.79	9.35
	1.25			48.06	-4.89	50.27	-1.28	54.34	4.80
	1.50			50.62	-7.44	53.54	-4.55	58.89	.24
	1.75			53.17	-10.00	56.81	-7.82	63.44	-4.31
7.00	--	46.61	-24.65						
	.75			48.68	-5.50	49.23	-.23	50.33	8.80
	1.00			51.23	-8.06	52.49	-3.50	54.89	4.25
	1.25			53.78	-10.61	55.76	-6.77	59.44	-.30
	1.50			56.34	-13.16	59.03	-10.04	63.99	-4.86
	1.75			58.89	-15.72	62.30	-13.31	68.54	-9.41
8.00	--	53.13	-31.16						
	.75			54.39	-11.22	54.72	-5.73	55.43	3.70
	1.00			56.95	-13.78	57.99	-8.99	59.98	-.85
	1.25			59.50	-16.33	61.25	-12.26	64.54	-5.40
	1.50			62.05	-18.88	64.52	-15.53	69.09	-9.96
	1.75			64.61	-21.44	67.79	-18.80	73.64	-14.51

[a]Temperature is held constant at the overall mean.

[b]The soilage and corn quantities are the same as derived in Table 34.

[c]The supplement in Table 3 is fed at the rate of .2 of a pound per day.

[d]The feeder steer is valued at $25.00/cwt.

[e]Derived from equation 45.

[f]The total feed cost includes the cost of corn and soilage plus 28 pounds of supplement valued at $3.50/cwt.

Table 46. (Continued)

Feed consumption (lbs.)	Ration: 8:1	5:1	3:1	2:1
Soilage	9,698	8,513	7,106	5,995
Corn	1,212	1,703	2,369	2,997
Supplement	28	28	28	28
Cost of feeder steer	$212.50	$212.50	$212.50	$212.50
Final weight		1,177	1,225	1,267
Grade	Av. good	High good	High good	Low choice
Selling price	$24.43	$24.99	$25.65	$26.17
Total revenue	$278.20	$294.18	$314.21	$331.44

Price of soilage ($/ton)	Price of corn ($/bu.)	8:1 Total feed cost ($)	8:1 Net revenue ($)	5:1 Total feed cost ($)	5:1 Net revenue ($)	3:1 Total feed cost ($)	3:1 Net revenue ($)	2:1 Total feed cost ($)	2:1 Net revenue ($)
1.00	--								
	.75	22.06	43.64	28.04	53.65	36.26	65.46	44.12	74.82
	1.00	27.48	38.23	35.64	46.05	46.83	54.88	57.50	61.44
	1.25	32.89	32.82	43.24	38.44	57.41	44.31	70.88	48.05
	1.50	38.30	27.40	50.84	30.84	67.98	33.73	84.26	34.68
	1.75	43.71	21.99	58.44	23.24	78.55	23.16	97.64	21.30
2.00	--								
	.75	26.91	38.79	32.29	49.39	39.81	61.90	47.12	71.82
	1.00	32.33	33.38	39.89	41.79	50.38	51.33	60.50	58.44
	1.25	37.74	27.97	47.49	34.19	60.96	40.75	73.88	45.06
	1.50	43.15	22.56	55.10	26.59	71.53	30.18	87.26	31.68
	1.75	48.56	17.14	62.70	18.99	82.11	19.60	100.64	18.30
3.00	--								
	.75	31.76	33.94	36.55	45.13	43.36	58.35	50.12	68.82
	1.00	37.17	28.53	44.15	37.53	53.94	47.78	63.50	55.44
	1.25	42.59	23.12	51.75	29.93	64.51	37.20	76.88	42.06
	1.50	48.00	17.71	59.35	22.33	75.09	26.63	90.26	28.68
	1.75	53.41	12.29	66.95	14.73	85.66	16.05	103.64	15.30
4.00	--								
	.75	36.61	29.09	40.81	40.88	[illegible]	54.90	53.11	65.93

	1.75	53.41	12.29	66.95	14.73	85.66	16.05	103.64	15.30
4.00	--								
	.75	36.61	29.09	40.81	40.88	46.92	54.80	53.11	65.83
	1.00	42.02	23.68	48.41	33.28	57.49	44.22	66.49	52.45
	1.25	47.44	18.27	56.01	25.68	68.06	33.65	79.87	39.07
	1.50	52.85	12.86	63.61	18.08	78.64	23.07	93.26	25.68
	1.75	58.26	7.45	71.21	10.47	89.21	12.50	106.64	12.30
5.00	--								
	.75	41.46	24.24	45.06	36.62	50.47	51.24	56.11	62.83
	1.00	46.87	18.83	52.66	27.02	61.04	40.61	67.47	49.45
	1.25	52.28	13.42	60.26	21.42	71.62	30.09	82.87	36.07
	1.50	57.70	8.01	67.86	13.82	82.19	19.52	96.25	22.69
	1.75	63.11	2.60	75.46	6.22	92.77	8.95	109.63	9.31
6.00	--								
	.75	46.31	19.39	49.32	32.36	54.02	47.69	59.11	59.83
	1.00	51.72	13.98	56.92	24.76	64.60	37.12	72.49	46.45
	1.25	57.13	8.57	64.52	17.16	75.18	26.54	85.87	33.07
	1.50	62.55	3.16	72.12	9.56	85.74	15.97	99.25	19.69
	1.75	79.96	-2.25	79.72	1.96	96.32	5.39	112.63	6.31
7.00	--								
	.75	51.16	14.55	53.58	28.11	57.57	44.14	62.10	56.84
	1.00	56.57	9.13	61.18	20.51	68.15	33.57	75.49	43.45
	1.25	61.98	3.72	68.78	12.91	78.72	22.99	88.87	30.07
	1.50	67.39	-1.69	76.38	5.31	89.30	12.41	102.25	16.69
	1.75	72.81	-7.10	83.98	-2.29	99.87	1.84	115.63	3.31
8.00	--								
	.75	56.01	9.70	57.83	23.85	61.13	40.58	65.10	53.84
	1.00	61.42	4.28	65.43	16.25	71.70	30.01	78.48	40.46
	1.25	66.83	-1.13	73.03	8.65	82.28	19.44	91.86	27.08
	1.50	72.24	-6.54	80.63	1.05	92.85	8.86	105.25	13.69
	1.75	77.66	-11.95	88.23	-6.55	103.43	-1.71	118.63	.31

Table 47. Predicted total feed consumption, total weight, grade, selling price, total revenue, total feed costs, and net revenue for good-to-choice feeder steers, weighing 85 pounds at the outset, for eight selected soilage-corn stilbestrol rations fed for 130 days (equation 58)[a]

Feed consumption[b] (lbs.)		Ration: All soilage		20:1		15:1		10:1	
Soilage		12,010		10,572		10,158		9,443	
Corn		--		529		677		944	
Supplement[c]		26		26		26		26	
Cost of feeder steer[d]		$212.50		$212.50		$212.50		$212.50	
Final weight		1,021		1,072		1,085		1,108	
Grade[e]		Av. Standard		High Standard		Low Good		Low Good	
Selling price		$22.90		$23.56		$23.75		$24.08	
Total revenue		$233.68		$252.55		$257.75		$266.86	
Price of soilage ($/ton)	Price of corn ($/bu.)	Total feed cost[f] ($)	Net revenue ($)	Total feed cost ($)	Net revenue ($)	Total feed cost ($)	Net revenue ($)	Total feed cost ($)	Net revenue ($)
1.00	--	6.92	14.26						
	.75			13.28	26.77	15.06	30.19	18.28	36.08
	1.00			15.64	24.41	18.08	27.17	22.49	31.87
	1.25			18.00	22.05	21.11	24.14	26.71	27.65
	1.50			20.36	19.69	24.13	21.12	30.93	23.43
	1.75			22.71	17.34	27.15	18.10	35.14	19.22
2.00	--	12.92	8.26						
	.75			18.56	21.49	20.14	25.11	23.00	31.36
	1.00			20.92	19.13	23.16	22.09	27.22	27.14
	1.25			23.28	16.77	26.18	19.07	31.43	22.93
	1.50			25.64	14.41	29.21	16.04	35.65	18.71
	1.75			28.00	12.05	32.23	13.02	39.86	14.50
3.00	--	18.93	2.25						
	.75			23.85	16.20	25.22	20.03	27.72	26.64
	1.00			26.21	13.84	28.24	17.01	31.94	22.43
	1.25			28.57	11.48	31.26	13.99	36.15	18.21
	1.50			30.93	9.12	34.29	10.96	40.37	13.99
	1.75			33.29	6.76	37.31	7.94	44.58	9.78
4.00	--	24.93	-3.75						
	.75			29.13	10.92	30.30	14.95	32.44	21.92
	1.00			31.49	8.56	33.32	11.93	36.66	17.70
	1.25			33.85	6.20	36.34	8.91	40.88	13.48
	1.50			36.21	3.84	39.37	5.88	45.09	9.27
	1.75			38.57	1.48	42.39	2.86	49.31	5.05

4.00	--	24.93	-3.75						
	.75			29.13	10.92	30.30	14.95	32.44	21.92
	1.00			31.49	8.56	33.32	11.93	36.66	17.70
	1.25			33.85	6.20	36.34	8.91	40.88	13.48
	1.50			36.21	3.84	39.37	5.88	45.09	9.27
	1.75			38.57	1.48	42.39	2.86	49.31	5.05
5.00	--	30.94	-9.76						
	.75			34.42	5.63	35.37	9.87	37.17	17.19
	1.00			36.78	3.27	38.40	6.85	41.38	12.98
	1.25			39.14	.91	41.42	3.83	45.60	8.76
	1.50			41.50	-1.45	44.44	.81	49.81	4.55
	1.75			43.86	-3.81	47.47	-2.22	54.03	.33
6.00	--	36.94	-15.76						
	.75			39.71	.34	40.45	4.80	41.89	12.47
	1.00			42.07	-2.02	43.48	1.77	46.10	8.26
	1.25			44.43	-4.38	46.50	-1.25	50.32	4.04
	1.50			46.79	-6.74	49.52	-4.27	54.53	-.17
	1.75			49.15	-9.10	52.55	-7.30	58.75	-4.39
7.00	--	42.95	-21.77						
	.75			44.99	-4.94	45.53	-.28	46.61	7.75
	1.00			47.35	-7.30	48.56	-3.31	50.82	3.54
	1.25			49.71	-9.66	51.58	-6.33	55.04	-.68
	1.50			52.07	-12.02	54.60	-9.35	59.26	-4.90
	1.75			54.43	-14.38	57.63	-12.38	63.47	-9.99
8.00	--	48.95	-27.77						
	.75			50.28	-10.23	50.61	-5.36	51.33	3.03
	1.00			52.64	-12.59	53.64	-8.39	55.55	-1.19
	1.25			55.00	-14.95	56.66	-11.41	59.76	-5.40
	1.50			57.36	-17.31	59.68	-14.42	63.98	-9.62
	1.75			59.72	-19.67	62.70	-17.45	68.19	-13.83

[a]Temperature is held constant at the overall mean.

[b]The soilage and corn quantities are the same as derived in Table 34.

[c]The supplement in Table 3 is fed at the rate of .2 of a pound per day.

[d]The feeder steer is valued at $25.00/cwt.

[e]Derived from equation 45.

[f]The total feed cost includes the cost of corn and soilage plus 26 pounds of supplement valued at $3.50/cwt.

Table 47. (Continued)

Feed consumption (lbs.)		Ration: 8:1		5:1		3:1		2:1	
Soilage		8,986		7,896		6,594		5,558	
Corn		1,123		1,579		2,198		2,779	
Supplement		26		26		26		26	
Cost of feeder steer		$212.50		$212.50		$212.50		$212.50	
Final weight		1,123		1,158		1,203		1,241	
Grade		Av. Good		Av. Good		High Good		Low Choice	
Selling price		$24.30		$24.80		$25.41		$25.89	
Total revenue		$272.78		$287.25		$305.50		$321.26	
Price of soilage ($/ton)	Price of corn ($/bu.)	Total feed cost ($)	Net revenue ($)	Total feed cost ($)	Net revenue ($)	Total feed cost ($)	Net revenue ($)	Total feed cost ($)	Net revenue ($)
1.00	--								
	.75	20.45	39.83	26.01	48.74	33.65	59.35	40.91	67.85
	1.00	25.46	34.82	33.06	41.69	43.46	49.54	53.31	55.45
	1.25	30.47	29.81	40.11	34.64	53.27	39.73	65.72	43.04
	1.50	35.49	24.79	47.16	27.59	63.09	29.91	78.12	30.64
	1.75	40.50	19.78	54.21	20.54	72.90	20.10	90.53	18.23
2.00	--								
	.75	24.94	35.34	29.96	44.79	36.94	56.06	43.68	65.08
	1.00	29.95	30.33	37.01	37.74	46.76	46.24	56.09	52.67
	1.25	34.97	25.31	44.06	30.69	56.57	36.43	68.50	40.26
	1.50	39.98	20.30	51.11	23.64	66.38	26.62	80.90	27.86
	1.75	45.00	15.28	58.16	16.59	76.20	16.80	93.31	15.45
3.00	--								
	.75	29.43	30.85	33.91	40.84	40.24	52.76	46.46	62.30
	1.00	34.44	25.84	40.96	33.79	50.05	42.95	58.87	49.89
	1.25	39.46	20.82	48.01	26.74	59.87	33.13	71.27	37.49
	1.50	44.47	15.81	55.06	19.69	69.68	23.32	83.68	25.08
	1.75	49.49	10.79	62.11	12.64	79.49	13.51	96.09	12.67
4.00	--								
	.75	33.92	26.36	37.85	36.90	43.54	49.46	49.24	59.52
	1.00	38.94	21.34	44.90	29.85	53.35	39.65	61.65	47.11
	1.25	[illegible]	[illegible]	[illegible]	[illegible]				

4.00	--								
	.75	33.92	26.36	37.85	36.90	43.54	49.46	49.24	59.52
	1.00	38.94	21.34	44.90	29.85	53.35	39.65	61.65	47.11
	1.25	43.95	16.33	51.95	22.80	63.16	29.84	74.05	34.71
	1.50	48.97	11.31	59.00	15.75	72.98	20.02	86.46	22.30
	1.75	53.98	6.30	66.05	8.70	82.79	10.21	98.86	9.90
5.00	--								
	.75	38.42	21.86	41.80	32.95	46.84	46.16	52.02	56.74
	1.00	43.43	16.85	48.85	25.90	56.65	36.35	64.43	44.33
	1.25	48.45	11.83	55.90	18.85	66.46	26.54	76.83	31.93
	1.50	53.46	6.82	62.95	11.80	76.27	16.73	89.24	19.52
	1.75	58.48	1.80	70.00	4.75	86.09	6.91	101.64	7.12
6.00	--								
	.75	42.91	17.37	45.75	29.00	50.13	42.87	54.80	53.96
	1.00	47.92	12.36	52.80	21.95	59.95	33.05	67.21	41.55
	1.25	52.94	7.34	59.85	14.90	69.76	23.24	79.61	29.15
	1.50	57.95	2.33	66.90	7.85	79.57	13.43	92.02	16.74
	1.75	62.97	-2.69	73.95	.80	89.39	3.61	104.42	4.34
7.00	--								
	.75	47.40	12.88	49.70	25.05	53.43	39.57	57.58	51.18
	1.00	52.42	7.86	56.75	18.00	63.24	29.76	69.98	38.78
	1.25	57.43	2.85	63.80	10.95	73.06	19.94	82.39	26.37
	1.50	62.45	-2.17	70.85	3.90	82.87	10.13	94.80	13.96
	1.75	67.46	-7.18	77.90	-3.15	92.68	.32	107.20	1.56
8.00	--								
	.75	51.90	8.38	53.65	21.10	56.73	36.27	60.36	48.40
	1.00	56.91	3.37	60.70	14.05	66.54	26.46	72.76	36.00
	1.25	61.93	-1.65	67.75	7.00	76.35	16.65	85.17	23.59
	1.50	66.94	-6.66	74.80	-.05	86.17	6.83	97.57	11.19
	1.75	71.95	-11.67	81.85	-7.10	95.98	-2.98	109.98	-1.22

Table 48. Predicted total feed consumption, total weight, grade, selling price, total revenue, total feed costs, and net revenue for good-to-choice feeder steers, weighing 850 pounds at the outset, for eight selected soilage-corn stilbestrol rations fed for 120 days (equation 58)[a]

Feed consumption[b] (lbs.)		Ration: All soilage		Ration: 20:1		Ration: 15:1		Ration: 10:1	
Soilage		10,997		9,713		9,339		8,692	
Corn		--		486		623		869	
Supplement[c]		24		24		24		24	
Cost of feeder steer[d]		$212.50		$212.50		$212.50		$212.50	
Final weight		1,015		1,060		1,072		1,093	
Grade[e]		Av. Standard		High Standard		Low Good		Low Good	
Selling price		$22.94		$23.53		$23.70		$24.00	
Total revenue		$232.74		$249.45		$254.08		$262.21	
Price of soilage ($/ton)	Price of corn ($/bu.)	Total feed cost[f] ($)	Net revenue ($)	Total feed cost ($)	Net revenue ($)	Total feed cost ($)	Net revenue ($)	Total feed cost ($)	Net revenue ($)
1.00	--	6.34	13.90						
	.75			12.20	24.74	13.85	27.73	16.83	32.88
	1.00			14.37	22.58	16.63	24.95	20.71	29.00
	1.25			16.54	20.41	19.41	22.17	24.59	25.12
	1.50			18.70	18.24	22.19	19.39	28.47	21.24
	1.75			20.87	16.07	24.97	16.61	32.35	17.36
2.00	--	11.84	8.40						
	.75			17.06	19.89	18.52	23.06	21.17	28.54
	1.00			19.22	17.72	21.30	20.28	25.05	24.66
	1.25			21.39	15.55	24.08	17.50	22.93	20.78
	1.50			23.56	13.38	26.86	14.72	32.81	16.90
	1.75			25.73	11.22	29.64	11.94	36.70	13.02
3.00	--	17.34	2.91						
	.75			21.91	15.03	23.19	18.39	25.52	24.19
	1.00			24.08	12.86	25.97	15.61	29.40	20.31
	1.25			26.25	10.70	28.75	12.83	33.28	16.43
	1.50			28.42	8.53	31.53	10.05	37.16	12.55
	1.75			30.59	6.36	34.31	7.27	41.04	8.67
4.00	--	22.83	-2.59						
	.75			26.77	10.18	27.86	13.72	29.87	19.85

	1.50			28.42	8.53	31.53	10.05	37.16	12.55
	1.75			30.59	6.36	34.31	7.27	41.04	8.67
4.00	--	22.83	-2.59						
	.75			26.77	10.18	27.86	13.72	29.87	19.85
	1.00			28.94	8.01	30.64	10.94	33.75	15.96
	1.25			31.11	5.84	33.42	8.16	37.63	12.08
	1.50			33.27	3.67	36.20	5.38	41.51	8.20
	1.75			35.44	1.50	38.98	2.60	45.39	4.32
5.00	--	28.33	-8.09						
	.75			31.63	5.32	32.53	9.05	34.21	15.50
	1.00			33.79	3.15	36.31	6.27	38.09	11.62
	1.25			35.96	.98	38.09	3.49	41.97	7.74
	1.50			38.13	-1.18	40.87	.71	45.85	3.86
	1.75			40.30	-3.35	43.65	-2.07	49.73	-.02
6.00	--	33.83	-13.59						
	.75			36.48	.46	37.20	4.38	38.56	11.15
	1.00			38.65	-1.71	39.98	1.60	42.44	7.27
	1.25			40.82	-3.87	42.76	-1.18	46.32	3.39
	1.50			42.99	-6.04	45.54	-3.96	50.20	-.49
	1.75			45.15	-8.21	48.32	-6.74	54.08	-4.37
7.00	--	39.33	-19.09						
	.75			41.34	-4.39	41.87	-.29	42.90	6.81
	1.00			43.51	-6.56	44.65	-3.07	46.78	2.93
	1.25			45.67	-8.73	47.43	-5.85	50.66	-.95
	1.50			47.84	-10.90	50.21	-8.63	54.55	-4.83
	1.75			50.01	-13.07	52.98	-11.41	58.43	-8.71
8.00	--	44.83	-24.59						
	.75			46.20	-9.25	46.54	-4.96	47.25	2.46
	1.00			48.36	-11.42	49.32	-7.74	51.13	-1.42
	1.25			50.53	-13.59	52.10	-10.52	55.01	-5.30
	1.50			52.70	-15.75	54.87	-13.30	58.89	-9.18
	1.75			54.87	-17.92	57.65	-16.08	62.77	-13.06

[a]Temperature is held constant at the overall mean.

[b]The soilage and corn quantities are the same as derived in Table 34.

[c]The supplement in Table 3 is fed at the rate of .2 of a pound per day.

[d]The feeder steer is valued at $25.00/cwt.

[e]Derived from equation 45.

[f]The total feed cost includes the cost of corn and soilage plus 24 pounds of supplement valued at $3.50/cwt.

Table 48. (Continued)

Feed consumption (lbs.)		Ration: 8:1		Ration: 5:1		Ration: 3:1		Ration: 2:1	
Soilage		8,276		7,281		6,083		5,122	
Corn		1,035		1,456		2,028		2,561	
Supplement		24		24		24		24	
Cost of feeder steer		$212.50		$212.50		$212.50		$212.50	
Final weight		1,106		1,138		1,179		1,215	
Grade		Low Good		Av. Good		High Good		Low Choice	
Selling price		$24.19		$24.64		$25.19		$25.64	
Total revenue		$267.52		$280.55		$297.09		$311.43	
Price of soilage ($/ton)	Price of corn ($/bu.)	Total feed cost ($)	Net revenue ($)	Total feed cost ($)	Net revenue ($)	Total feed cost ($)	Net revenue ($)	Total feed cost ($)	Net revenue ($)
1.00	--								
	.75	18.83	36.19	23.98	44.07	31.04	53.55	37.70	61.22
	1.00	23.45	31.57	30.49	37.57	40.09	44.50	49.14	49.79
	1.25	28.07	26.95	36.99	31.07	49.15	35.45	60.57	38.36
	1.50	32.69	22.33	43.49	24.57	58.20	26.39	72.00	26.92
	1.75	37.31	17.71	49.99	18.06	67.25	17.34	83.44	15.49
2.00	--								
	.75	22.97	32.05	27.62	40.43	34.08	50.51	40.26	58.66
	1.00	27.59	27.43	34.13	33.93	43.13	41.46	51.70	47.23
	1.25	32.21	22.81	40.63	27.43	52.19	32.41	63.13	35.80
	1.50	36.83	18.19	47.13	20.92	61.24	23.35	74.56	24.36
	1.75	41.45	13.57	53.63	14.42	70.29	14.30	86.00	12.93
3.00	--								
	.75	27.11	27.91	31.27	36.79	37.12	47.47	42.82	56.10
	1.00	31.73	23.29	37.77	30.29	46.18	38.42	54.26	44.67
	1.25	36.35	18.67	44.27	23.79	55.23	29.36	65.69	33.23
	1.50	40.97	14.05	50.77	17.28	64.28	20.31	77.12	21.80
	1.75	45.58	9.44	57.27	10.78	73.33	11.26	88.56	10.37
4.00	--								
	.75	31.25	23.77	34.91	33.15	40.17	44.43	45.38	53.54
	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

4.00	--								
	.75	31.25	23.77	34.91	33.15	40.17	44.43	45.38	53.54
	1.00	35.87	19.15	41.41	26.65	49.22	35.37	56.82	42.11
	1.25	40.49	14.54	47.91	20.14	58.27	26.32	68.25	30.67
	1.50	45.10	9.92	54.41	13.64	67.32	17.27	79.69	19.24
	1.75	49.72	5.30	60.91	7.14	76.38	8.22	91.12	7.81
5.00	--								
	.75	35.39	19.63	38.55	29.51	43.21	41.39	47.95	50.98
	1.00	40.00	15.02	45.05	23.01	52.26	32.33	59.38	39.55
	1.25	44.62	10.40	51.55	16.50	61.31	23.28	70.81	28.11
	1.50	49.24	5.78	58.05	10.00	70.37	14.23	82.25	16.68
	1.75	53.86	1.16	64.55	3.50	79.42	5.17	93.68	5.25
6.00	--								
	.75	39.52	15.50	42.19	25.87	46.25	38.34	50.51	48.42
	1.00	44.14	10.88	48.69	19.36	55.30	29.29	61.94	36.99
	1.25	48.76	6.26	55.19	12.86	64.35	20.24	73.37	25.55
	1.50	53.38	1.64	61.69	6.36	73.41	11.19	84.81	14.12
	1.75	58.00	-2.98	68.19	-.14	82.46	2.13	96.24	2.68
7.00	--								
	.75	43.66	11.36	45.83	22.22	49.29	35.30	53.07	45.86
	1.00	48.28	6.74	52.33	15.72	58.34	26.25	64.50	34.42
	1.25	52.90	2.12	58.83	9.22	67.40	17.20	75.94	22.99
	1.50	57.52	-2.50	65.33	2.72	76.45	8.14	87.37	11.56
	1.75	62.14	-7.12	71.83	-3.78	85.50	-.91	98.80	.12
8.00	--								
	.75	47.80	7.22	49.47	18.58	52.33	32.26	55.63	43.30
	1.00	52.42	2.60	55.97	12.08	61.38	23.21	67.06	31.86
	1.25	57.04	-2.02	62.47	5.58	70.44	14.15	78.50	20.43
	1.50	61.66	-6.64	68.97	-.92	79.49	5.10	89.93	9.00
	1.75	66.27	-11.25	75.47	-7.42	88.54	-3.95	101.36	-2.44

Table 49. Predicted total feed consumption, total weight, grade, selling price, total revenue, total feed costs, and net revenue for good-to-choice feeder steers, weighing 850 pounds at the outset, for eight selected soilage-corn stilbestrol rations fed for 90 days (equation 58)[a]

Feed consumption[b] (lbs.)	Ration: All soilage	Ration: 20:1	Ration: 15:1	Ration: 10:1
Soilage	8,048	7,180	6,919	6,463
Corn	--	359	461	646
Supplement[c]	18	18	18	18
Cost of feeder steer[d]	$212.50	$212.50	$212.50	$212.50
Final weight	988	1,019	1,027	1,042
Grade[e]	Av. Standard	High Standard	High Standard	Low Good
Selling price	$23.19	$23.57	$23.69	$23.89
Total revenue	$229.13	$240.24	$243.37	$248.92

Price of soilage ($/ton)	Price of corn ($/bu.)	All soilage: Total feed cost[f] ($)	All soilage: Net revenue ($)	20:1: Total feed cost ($)	20:1: Net revenue ($)	15:1: Total feed cost ($)	15:1: Net revenue ($)	10:1: Total feed cost ($)	10:1: Net revenue ($)
1.00	--	4.65	11.97						
	.75			9.03	18.71	10.27	20.60	12.52	23.90
	1.00			10.63	17.11	12.33	18.54	15.40	21.02
	1.25			12.23	15.51	14.39	16.48	18.29	18.13
	1.50			13.84	13.91	16.45	14.42	21.17	15.25
	1.75			15.44	12.30	18.51	12.36	24.06	12.36
2.00	--	8.68	7.95						
	.75			12.62	15.12	13.73	17.14	15.75	20.67
	1.00			14.22	13.52	15.79	15.08	18.63	17.79
	1.25			15.82	11.92	17.85	13.02	21.52	14.90
	1.50			17.43	10.32	19.91	10.96	24.40	12.02
	1.75			19.03	8.71	21.97	8.90	27.29	9.13
3.00	--	12.70	3.92						
	.75			16.21	11.53	17.19	13.68	18.98	17.44
	1.00			17.81	9.93	19.25	11.62	21.86	14.56
	1.25			19.41	8.33	21.31	9.56	22.75	11.67
	1.50			21.02	6.73	23.37	7.50	27.63	8.79

	1.25			19.41	8.33	21.31	9.56	22.75	11.67
	1.50			21.02	6.73	23.37	7.50	27.63	8.79
	1.75			22.62	5.12	25.42	5.44	30.52	5.90
4.00	--	16.73	-.10						
	.75			19.80	7.94	20.65	10.22	22.21	14.21
	1.00			21.40	6.34	22.71	8.16	25.10	11.33
	1.25			23.00	4.74	24.77	6.10	27.98	8.44
	1.50			24.61	3.14	26.83	4.04	30.87	5.56
	1.75			26.21	1.53	28.88	1.98	33.75	2.67
5.00	--	20.75	-4.12						
	.75			23.39	4.35	24.11	6.76	25.44	10.98
	1.00			24.99	2.75	26.17	4.70	28.33	8.09
	1.25			26.59	1.15	28.23	2.64	31.21	5.21
	1.50			28.20	-.45	30.28	.58	34.10	2.32
	1.75			29.80	-2.06	32.34	-1.48	36.98	-.56
6.00	--	24.77	-8.15						
	.75			26.98	.76	27.57	3.30	28.67	7.75
	1.00			28.58	-.84	29.63	1.24	31.56	4.86
	1.25			30.18	-2.44	31.69	-.82	34.44	1.98
	1.50			31.79	-4.04	33.74	-2.88	37.33	-.91
	1.75			33.39	-5.65	35.80	-4.94	40.21	-3.79
7.00	--	28.80	-12.17						
	.75			30.57	-2.83	31.03	-.16	31.90	4.52
	1.00			32.17	-4.43	33.09	-2.22	34.79	1.63
	1.25			33.77	-6.03	35.15	-4.28	37.67	-1.25
	1.50			35.38	-7.63	37.20	-6.34	40.56	-4.14
	1.75			36.98	-9.24	39.26	-8.40	43.45	-7.02
8.00	--	32.82	-16.20						
	.75			34.16	-6.42	34.49	-3.62	35.14	1.29
	1.00			35.76	-8.02	36.55	-5.68	38.02	-1.60
	1.25			37.36	-9.62	38.60	-7.74	40.91	-4.49
	1.50			38.97	-11.22	40.66	-9.80	43.79	-7.37
	1.75			40.57	-12.83	42.72	-11.86	46.68	-10.26

[a]Temperature is held constant at the overall mean.

[b]The soilage and corn quantities are the same as derived in Table 34.

[c]The supplement in Table 3 is fed at the rate of .2 of a pound per day.

[d]The feeder steer is valued at $25.00/cwt.

[e]Derived from equation 45.

[f]The total feed cost includes the cost of corn and soilage plus 18 pounds of supplement valued at $3.50/cwt.

Table 49. (Continued)

Feed consumption (lbs.)		Ration: 8:1		5:1		3:1		2:1	
Soilage		6,166		5,444		4,554		3,824	
Corn		771		1,089		1,518		1,912	
Supplement		18		18		18		18	
Cost of feeder steer		$212.50		$212.50		$212.50		$212.50	
Final weight		1,051		1,075		1,105		1,132	
Grade		Low Good		Low Good		Av. Good		High Good	
Selling price		$24.02		$24.35		$24.74		$25.07	
Total revenue		$252.59		$261.71		$273.49		$283.79	
Price of soilage ($/ton)	Price of corn ($/bu.)	Total feed cost ($)	Net revenue ($)	Total feed cost ($)	Net revenue ($)	Total feed cost ($)	Net revenue ($)	Total feed cost ($)	Net revenue ($)
1.00	--								
	.75	14.03	26.05	17.93	31.28	23.24	37.76	28.15	43.14
	1.00	17.48	22.61	22.79	26.42	30.02	30.98	36.69	34.60
	1.25	20.92	19.17	27.65	21.56	36.79	24.20	45.22	26.07
	1.50	24.36	15.73	32.51	16.70	43.57	17.42	53.76	17.53
	1.75	27.80	12.29	37.38	11.84	50.35	10.65	62.30	8.99
2.00	--								
	.75	17.12	22.97	20.66	28.56	25.52	35.48	30.06	41.23
	1.00	20.56	19.53	25.52	23.70	32.29	28.70	38.60	32.69
	1.25	24.00	16.09	30.38	18.84	39.07	21.92	47.14	24.15
	1.50	27.44	12.65	35.24	13.98	45.85	15.15	55.67	15.62
	1.75	30.88	9.21	40.10	9.12	52.62	8.37	64.21	7.08
3.00	--								
	.75	20.20	19.89	23.38	25.84	27.79	33.20	31.97	39.31
	1.00	23.64	16.45	28.24	20.98	34.57	26.42	40.51	30.78
	1.25	27.08	13.01	33.10	16.12	41.35	19.65	49.05	22.24
	1.50	30.52	9.57	37.96	11.26	48.12	12.87	57.58	13.71
	1.75	33.96	6.13	42.82	6.39	54.90	6.09	66.12	5.17
4.00	--								

	1.50	30.52	9.57	37.96	11.26	48.12	12.87	57.58	13.71
	1.75	33.96	6.13	42.82	6.39	54.90	6.09	66.12	5.17
4.00	--								
	.75	23.28	16.80	26.10	23.11	30.07	30.92	33.89	37.40
	1.00	26.72	13.36	30.96	18.25	36.85	24.15	42.42	28.87
	1.25	30.16	9.92	35.82	13.39	43.62	17.37	50.96	20.33
	1.50	33.61	6.48	40.68	8.53	50.40	10.59	59.50	11.79
	1.75	37.05	3.04	45.54	3.67	57.18	3.81	68.03	3.26
5.00	--								
	.75	26.37	13.72	28.82	20.39	32.35	28.65	35.80	35.49
	1.00	29.81	10.28	33.68	15.53	39.12	21.87	44.34	26.95
	1.25	33.25	6.84	38.54	10.67	45.90	15.09	52.87	18.42
	1.50	36.69	3.40	43.40	5.81	52.68	8.31	61.41	9.88
	1.75	40.13	-.04	48.26	.95	59.46	1.54	69.94	1.35
6.00	--								
	.75	29.45	10.64	31.54	17.67	34.62	26.37	37.71	33.58
	1.00	32.89	7.20	36.40	12.81	41.40	19.59	46.25	25.04
	1.25	36.33	3.76	41.26	7.95	48.18	12.81	54.78	16.51
	1.50	39.77	.32	46.12	3.09	54.96	6.04	63.32	7.97
	1.75	43.21	-3.12	50.98	-1.77	61.73	-.74	71.86	-.57
7.00	--								
	.75	32.53	7.56	34.26	14.95	36.90	24.09	39.62	31.67
	1.00	35.97	4.12	39.12	10.09	43.68	17.32	48.16	23.13
	1.25	39.41	.68	43.99	5.23	50.46	10.54	56.70	14.59
	1.50	42.85	-2.77	48.85	.37	57.23	3.76	65.23	6.06
	1.75	46.29	-6.21	53.71	-4.49	64.01	-3.02	73.77	-2.48
8.00	--								
	.75	35.61	4.47	36.99	12.23	39.18	21.82	41.54	29.75
	1.00	39.06	1.03	41.85	7.37	45.96	15.04	50.07	21.22
	1.25	42.50	-2.41	46.71	2.51	52.73	8.26	58.61	12.68
	1.50	45.94	-5.85	51.57	-2.35	59.51	1.48	67.14	4.15
	1.75	49.38	-9.29	56.43	-7.21	66.29	-5.29	75.68	4.39

the cost of the supplement valued at \$3.50 per hundredweight. The profit above feed costs from feeding the 20:1 ration for 140 days is \$20.54. All of the other rations and feed price combinations are interpreted in a similar manner. The expected profits from feeding various non-stilbestrol rations for 140, 130, 120 and 90 days with various feed price assumptions are presented in Tables 50, 51, 52 and 53, respectively.

With most of the feed price combinations, the greatest profits are obtained when the heaviest corn ration is fed. However, when the price of soilage is low relative to the price of corn, then the most profitable ration is a ration of less corn and more soilage.

While Tables 46 through 53 show what the expected profits are from feeding various soilage-corn rations, for various periods of time with various feed price combinations, they do not show at least not very clearly what the optimum feeding period would be for any given ration and feed price combination.

The profit functions shown in equations 58 and 59 can be written in general terms as:

$$(60) \quad \pi = (a_1 + a_2C + a_3F + a_4C^2 + a_5F^2 + a_6 + a_7H)(b_1 + b_2F + b_3T + b_4T^2 + b_5FT) - P_CC - P_FF - .2TP_S - K$$

where the a_i's ($i = 1, \ldots, 7$) refer to the constants in the

Table 50. Predicted total feed consumption, total weight, grade, selling price, total revenue, total feed costs, and net revenue for good-to-choice feeder steers, weighing 850 pounds at the outset, for eight selected soilage-corn non-stilbestrol rations fed for 140 days (equation 59)[a]

Feed consumption[b] (lbs.)	Ration: All soilage	Ration: 20:1	Ration: 15:1	Ration: 10:1
Soilage	15,240	13,094	12,487	11,438
Corn	--	655	832	1,144
Supplement[c]	28	28	28	28
Cost of feeder steer[d]	$212.50	$212.50	$212.50	$212.50
Final weight	1,040	1,090	1,103	1,124
Grade[e]	Av. Standard	High Standard	Low Good	Low Good
Selling price	$22.06	$23.01	$23.28	$23.75
Total revenue	$229.50	$250.84	$256.68	$266.80

Price of soilage ($/ton)	Price of corn ($/bu.)	All soilage: Total feed cost[f] ($)	All soilage: Net revenue ($)	20:1: Total feed cost ($)	20:1: Net revenue ($)	15:1: Total feed cost ($)	15:1: Net revenue ($)	10:1: Total feed cost ($)	10:1: Net revenue ($)
1.00	--	8.32	8.68						
	.75			16.02	22.32	18.09	26.09	21.74	32.57
	1.00			18.94	19.40	21.81	22.37	26.85	27.45
	1.25			21.86	16.48	25.53	18.65	31.95	22.35
	1.50			24.78	13.56	29.24	14.94	37.06	17.24
	1.75			27.71	10.63	32.96	11.22	42.16	12.14
2.00	--	15.94	1.06						
	.75			22.56	15.78	24.34	19.84	27.46	26.84
	1.00			25.48	12.86	28.05	16.13	32.56	21.74
	1.25			28.41	9.93	31.77	12.41	37.67	16.63
	1.50			31.33	7.01	35.49	8.69	42.78	11.52
	1.75			34.25	4.09	39.20	4.98	47.88	6.42
3.00	--	23.56	-6.56						
	.75			29.11	9.23	30.58	13.60	33.18	21.12
	1.00			32.03	6.31	34.30	9.88	38.28	16.02

	.75			29.11	9.23	30.58	13.60	33.18	21.12
	1.00			32.03	6.31	34.30	9.88	38.28	16.02
	1.25			34.95	3.39	38.01	6.17	43.39	10.91
	1.50			37.88	.46	41.73	2.45	48.50	5.80
	1.75			40.80	-2.46	45.45	-1.27	53.60	.70
4.00	--	31.18	-14.18						
	.75			35.66	2.68	36.82	7.36	38.90	15.40
	1.00			38.58	-.24	40.54	3.64	44.00	10.30
	1.25			41.50	-3.16	44.26	-.08	49.11	5.19
	1.50			44.42	-6.08	47.97	-3.79	54.22	.08
	1.75			47.35	-9.01	51.69	-7.51	59.32	-5.02
5.00	--	38.80	-21.80						
	.75			42.20	-3.86	43.07	1.11	44.62	9.68
	1.00			45.13	-6.79	46.78	-2.60	49.72	4.58
	1.25			48.05	-9.71	50.50	-6.32	54.83	-.53
	1.50			50.97	-12.63	54.22	-10.04	59.93	-5.63
	1.75			53.89	-15.55	57.93	-13.75	65.04	-10.74
6.00	--	46.42	-29.42						
	.75			48.75	-10.41	49.31	-5.13	50.33	3.97
	1.00			51.67	-13.33	53.02	-8.84	55.44	-1.14
	1.25			54.59	-16.25	56.74	-12.56	60.55	-6.25
	1.50			57.52	-19.18	60.46	-16.28	65.65	-11.35
	1.75			60.44	-22.10	64.18	-20.00	70.76	-16.46
7.00	--	54.04	-37.04						
	.75			55.30	-16.96	55.55	-11.37	56.05	-1.75
	1.00			58.22	-19.88	59.27	-15.09	61.16	-6.86
	1.25			61.14	-22.80	62.99	-18.81	66.27	-11.97
	1.50			64.06	-25.72	66.70	-22.52	71.37	-17.07
	1.75			66.99	-28.65	70.42	-26.24	76.48	-22.18
8.00	--	61.66	-44.66						
	.75			61.84	-23.50	61.80	-17.62	61.77	-7.47
	1.00			64.77	-26.43	65.51	-21.33	66.88	-12.58
	1.25			67.69	-29.35	69.23	-25.05	71.99	-17.69
	1.50			70.61	-32.27	72.95	-28.77	77.09	-22.79
	1.75			73.53	-35.19	76.66	-32.48	82.20	-27.90

[a]Temperature is held constant at the overall mean.

[b]The soilage and corn quantities are the same as derived in Table 35.

[c]The supplement in Table 3 is fed at the rate of .2 of a pound per day.

[d]The feeder steer is valued at $25.00/cwt.

[e]Derived from equation 46.

[f]The total feed cost includes the cost of corn and soilage plus 28 pounds of supplement valued at $2.50/cwt.

Table 50. (Continued)

Feed consumption (lbs.)	Ration 8:1	5:1	3:1	2:1
Soilage	10,768	9,176	7,296	5,827
Corn	1,346	1,835	2,432	2,914
Supplement	28	28	28	28
Cost of feeder steer	$212.50	$212.50	$212.50	$212.50
Final weight	1,137	1,166	1,199	1,222
Grade	Low Good	Av. Good	High Good	Low Choice
Selling price	$24.04	$24.75	$25.58	$26.23
Total revenue	$273.29	$288.66	$306.71	$320.64

Price of soilage ($/ton)	Price of corn ($/bu.)	8:1 Total feed cost ($)	8:1 Net revenue ($)	5:1 Total feed cost ($)	5:1 Net revenue ($)	3:1 Total feed cost ($)	3:1 Net revenue ($)	2:1 Total feed cost ($)	2:1 Net revenue ($)
1.00	--								
	.75	24.11	36.68	29.87	46.29	36.92	57.29	42.63	65.50
	1.00	30.12	30.67	38.06	38.10	47.77	46.44	55.64	52.50
	1.25	36.13	24.66	46.25	29.91	58.63	35.58	68.65	39.49
	1.50	42.14	18.65	54.44	21.72	69.49	24.72	81.66	26.48
	1.75	48.15	12.64	62.64	13.52	80.34	13.87	94.66	13.48
2.00	--								
	.75	29.49	31.30	34.45	41.71	40.57	53.64	45.55	62.59
	1.00	35.50	25.29	42.65	33.51	51.42	42.79	58.56	49.58
	1.25	41.51	19.28	50.84	25.32	62.28	31.93	71.56	36.58
	1.50	47.52	13.27	59.03	17.13	73.14	21.08	84.57	23.57
	1.75	53.53	7.26	67.23	8.93	83.99	10.22	97.58	10.56
3.00	--								
	.75	34.88	25.91	39.04	37.12	44.21	50.00	48.46	59.68
	1.00	40.89	19.90	47.24	28.92	55.07	39.14	61.47	46.67
	1.25	46.90	13.89	55.43	20.73	65.93	28.28	74.48	33.66
	1.50	52.90	7.89	63.62	12.54	76.78	17.43	87.48	20.66
	1.75	58.91	1.88	71.81	4.35	87.64	6.57	100.49	7.65
4.00	--								
	.75	40.26	20.53	43.63	32.53	47.86	46.35	51.38	56.76
	1.00	46.27	14.52	51.82	24.34	58.72	35.49	64.38	43.76
	1.25	52.28	8.51	60.02	16.14	69.58	24.63	77.39	30.75
	1.50	58.29	2.50	68.21	7.95	80.43	13.79	90.40	17.74
	1.75	64.30	-3.51	76.40	-.24	91.29	2.92	103.40	4.74

	.75	40.26	20.53	43.63	32.53	47.86	46.35	51.38	56.76
	1.00	46.27	14.52	51.82	24.34	58.72	35.49	64.38	43.76
	1.25	52.28	8.51	60.02	16.14	69.58	24.63	77.39	30.75
	1.50	58.29	2.50	68.21	7.95	80.43	13.78	90.40	17.74
	1.75	64.30	-3.51	76.40	-.24	91.29	2.92	103.40	4.74
5.00	--								
	.75	45.65	15.14	48.22	27.94	51.51	42.70	54.29	53.85
	1.00	51.65	9.14	56.41	19.75	62.37	31.84	67.30	40.84
	1.25	57.66	3.13	64.60	11.56	73.22	20.99	80.30	27.84
	1.50	63.67	-2.88	72.80	3.36	84.08	10.13	93.31	14.83
	1.75	69.68	-8.89	80.99	-4.83	94.94	-.73	106.32	8.82
6.00	--								
	.75	51.03	9.76	52.81	23.35	55.16	39.05	57.20	50.94
	1.00	57.04	3.75	61.00	15.16	66.01	28.20	70.21	37.93
	1.25	63.05	-2.26	69.19	6.97	76.87	17.34	83.22	24.92
	1.50	69.06	-8.27	77.38	-1.22	87.73	6.48	96.22	11.92
	1.75	75.06	-14.27	85.58	-9.42	98.58	-4.37	109.23	-1.09
7.00	--								
	.75	56.41	4.38	57.39	18.77	58.81	35.40	60.12	48.02
	1.00	62.42	-1.63	65.59	10.57	69.66	24.55	73.12	35.02
	1.25	68.43	-7.64	73.78	2.38	80.52	13.69	86.13	22.01
	1.50	74.44	-13.65	81.97	-5.81	91.38	2.83	99.14	9.00
	1.75	80.45	-19.66	90.17	-14.01	102.23	-8.02	112.14	-4.00
8.00	--								
	.75	61.80	-1.01	61.98	14.18	62.45	31.76	63.03	45.11
	1.00	67.81	-7.02	70.17	5.99	73.31	20.90	76.04	32.10
	1.25	73.81	-13.02	78.37	-2.21	84.17	10.04	89.04	19.10
	1.50	79.82	-19.03	86.56	-10.40	95.02	-.81	102.05	6.09
	1.75	85.83	-25.04	94.75	-18.59	105.88	-11.67	115.06	-6.92

Table 51. Predicted total feed consumption, total weight, grade, selling price, total revenue, total feed costs, and net revenue for good-to-choice feeder steers, weighing 850 pounds at the outset, for eight selected soilage-corn non-stilbestrol rations fed for 130 days (equation 59)[a]

Feed consumption[b] (lbs.)	Ration: All soilage	20:1	15:1	10:1
Soilage	14,096	12,112	11,550	10,579
Corn	--	606	770	1,057
Supplement[c]	26	26	26	26
Cost of feeder steer[d]	$212.50	$212.50	$212.50	$212.50
Final weight	1,035	1,081	1,093	1,113
Grade[e]	Av. Standard	High Standard	Low Good	Low Good
Selling price	$22.22	$23.04	$23.27	$23.68
Total revenue	$230.01	$249.17	$254.41	$263.48

Price of soilage ($/ton)	Price of corn ($/bu.)	All soilage: Total feed cost[f] ($)	All soilage: Net revenue ($)	20:1: Total feed cost ($)	20:1: Net revenue ($)	15:1: Total feed cost ($)	15:1: Net revenue ($)	10:1: Total feed cost ($)	10:1: Net revenue ($)
1.00	--	7.70	9.81						
	.75			14.82	21.85	16.74	25.17	20.11	30.87
	1.00			17.52	19.15	20.18	21.73	24.83	26.15
	1.25			20.22	16.45	23.61	18.30	29.55	21.43
	1.50			22.93	13.74	27.05	14.86	34.28	16.70
	1.75			25.63	11.04	30.49	11.42	39.00	11.98
2.00	--	14.75	2.76						
	.75			20.87	15.80	22.51	19.40	25.40	25.58
	1.00			23.58	13.09	25.95	15.96	30.12	20.86
	1.25			26.28	10.39	29.39	12.52	34.84	16.13
	1.50			28.98	7.69	32.83	9.08	39.57	11.41
	1.75			31.69	4.98	36.26	5.65	44.29	6.69
3.00	--	21.79	-4.28						
	.75			26.93	9.74	28.29	13.62	30.69	20.29
	1.00			29.63	7.04	31.73	10.18	35.41	15.57
	1.25			32.34	4.33	35.16	6.75	40.13	10.85
	1.50			35.04	1.63	38.60	3.31	44.85	6.13
	1.75			37.74	-1.07	42.04	-.13	49.58	1.40
4.00	--	28.84	-11.33						
	.75			32.98	3.69	34.06	7.85	35.98	15.00
	1.00			35.69	.98	37.50	4.41	40.70	10.28

	.75			32.98	3.69	34.06	7.85	35.98	15.00
	1.00			35.69	.98	37.50	4.41	40.70	10.28
	1.25			38.39	-1.72	40.94	.97	45.42	5.56
	1.50			41.10	-4.43	44.38	-2.47	50.14	.84
	1.75			43.80	-7.13	47.81	-5.90	54.87	-3.89
5.00	--	35.89	-18.38						
	.75			39.04	-2.37	39.84	2.07	41.27	9.71
	1.00			41.74	-5.07	43.28	-1.37	45.99	4.99
	1.25			44.45	-7.78	46.71	-4.80	50.71	.27
	1.50			47.15	-10.48	50.15	-8.24	55.43	-4.45
	1.75			49.86	-13.19	53.59	-11.68	60.16	-9.18
6.00	--	42.94	-25.43						
	.75			45.10	-8.43	45.61	-3.70	46.55	4.43
	1.00			47.80	-11.13	49.05	-7.14	51.28	-.30
	1.25			50.50	-13.83	52.49	-10.58	56.00	-5.02
	1.50			53.21	-16.54	55.93	-14.02	60.72	-9.74
	1.75			55.91	-19.24	59.36	-17.45	65.45	-14.47
7.00	--	49.99	-32.48						
	.75			51.15	-14.48	51.39	-9.48	51.84	-.86
	1.00			53.86	-17.19	54.83	-12.92	56.57	-5.59
	1.25			56.56	-19.89	58.26	-16.35	61.29	-10.31
	1.50			59.26	-22.59	61.70	-19.79	66.01	-15.03
	1.75			61.97	-25.30	65.14	-23.23	70.73	-19.75
8.00	--	57.03	-39.52						
	.75			57.21	-20.54	57.16	-15.25	57.13	-6.15
	1.00			59.91	-23.24	60.60	-18.69	61.86	-10.88
	1.25			62.62	-25.95	64.04	-22.13	66.58	-15.60
	1.50			65.32	-28.65	67.48	-25.57	71.30	-20.32
	1.75			68.02	-31.35	70.92	-29.01	76.02	-25.04

[a]Temperature is held constant at the overall mean.

[b]The soilage and corn quantities are the same as derived in Table 35.

[c]The supplement in Table 3 is fed at the rate of .2 of a pound per day.

[d]The feeder steer is valued at $25.00/cwt.

[e]Derived from equation 46.

[f]The total feed cost includes the cost of corn and soilage plus 26 pounds of supplement valued at $2.50/cwt.

Table 51. (Continued)

Feed consumption (lbs.)		Ration: 8:1		5:1		3:1		2:1	
Soilage		9,957		8,481		6,737		5,375	
Corn		1,245		1,696		2,246		2,687	
Supplement		26		26		26		26	
Cost of feeder steer		$212.50		$212.50		$212.50		$212.50	
Final weight		1,125		1,153		1,184		1,207	
Grade		Low Good		Av. Good		High Good		Low Choice	
Selling price		$23.93		$24.55		$25.27		$25.84	
Total revenue		$269.29		$283.07		$299.28		$311.80	
Price of soilage ($/ton)	Price of corn ($/bu.)	Total feed cost ($)	Net revenue ($)	Total feed cost ($)	Net revenue ($)	Total feed cost ($)	Net revenue ($)	Total feed cost ($)	Net revenue ($)
1.00	--								
	.75	22.30	34.49	27.61	42.96	34.09	52.69	39.33	59.97
	1.00	27.85	28.94	35.18	35.39	44.12	42.66	51.33	47.97
	1.25	33.41	23.38	42.75	27.82	54.14	32.64	63.33	35.97
	1.50	38.97	17.82	50.32	20.25	64.17	22.61	75.32	23.98
	1.75	44.52	12.27	57.90	12.67	74.19	12.59	87.32	11.98
2.00	--								
	.75	27.28	29.51	31.85	38.72	37.46	49.32	42.02	57.28
	1.00	32.83	23.96	39.42	31.15	47.49	39.29	54.01	45.29
	1.25	38.39	18.40	46.99	23.58	57.51	29.27	66.01	33.29
	1.50	43.95	12.84	54.56	16.01	67.53	19.25	78.01	21.29
	1.75	49.50	7.29	62.14	8.44	77.56	9.22	90.01	9.29
3.00	--								
	.75	32.25	24.54	36.09	34.48	40.83	45.95	44.70	54.60
	1.00	37.81	18.98	43.66	26.91	50.85	35.93	56.70	42.60
	1.25	43.37	13.42	51.23	19.34	60.88	25.90	68.70	30.60
	1.50	48.92	7.87	58.80	11.77	70.90	15.88	80.70	18.60
	1.75	54.48	2.31	66.38	4.19	80.93	5.85	92.69	6.61

	1.75	54.48	2.31	66.38	4.19	80.93	5.85	92.69	6.61
4.00	--								
	.75	37.23	19.56	40.33	30.24	44.20	42.58	47.39	51.91
	1.00	42.79	14.00	47.90	22.67	54.22	32.56	59.39	39.91
	1.25	48.35	8.44	55.47	15.10	64.25	22.53	71.39	27.91
	1.50	53.90	2.89	63.04	7.53	74.27	12.51	83.38	15.92
	1.75	59.46	-2.67	70.62	-.05	84.30	2.48	95.38	3.92
5.00	--								
	.75	42.21	14.58	44.57	26.00	47.57	39.21	50.08	49.22
	1.00	47.77	9.02	52.14	18.43	57.59	29.19	62.08	37.22
	1.25	53.32	3.47	59.71	10.86	67.61	19.17	74.07	25.23
	1.50	58.88	-2.09	67.28	3.29	77.64	9.14	86.07	13.23
	1.75	64.44	-7.65	74.86	-4.29	87.66	-.88	98.07	1.23
6.00	--								
	.75	47.19	9.60	48.81	21.76	50.93	35.85	52.77	46.53
	1.00	52.75	4.04	56.38	14.19	60.96	25.82	64.76	34.54
	1.25	58.30	-1.51	63.95	6.62	70.98	15.80	76.76	22.54
	1.50	63.86	-7.07	71.53	-.96	81.01	5.77	88.76	10.54
	1.75	69.42	-12.63	79.10	-8.53	91.03	-4.25	100.76	-1.46
7.00	--								
	.75	52.17	4.62	53.05	17.52	54.30	32.48	55.45	43.85
	1.00	57.73	-.94	60.62	9.95	64.33	22.45	67.45	31.85
	1.25	63.28	-6.49	68.19	2.38	74.35	12.43	79.45	19.85
	1.50	68.84	-12.05	75.77	-5.20	84.38	2.40	91.45	7.85
	1.75	74.39	-17.60	83.34	-12.77	94.40	-7.62	103.44	-4.14
8.00	--								
	.75	57.15	-.36	57.29	13.28	57.67	29.11	58.14	41.16
	1.00	62.70	-5.91	64.86	5.71	67.69	19.09	70.14	29.16
	1.25	68.26	-11.47	72.43	-1.86	77.72	9.06	82.14	17.16
	1.50	73.82	-17.03	80.01	-9.44	87.74	-.96	94.13	5.17
	1.75	79.37	-22.58	87.58	-17.01	97.77	-10.99	106.13	-6.83

Table 52. Predicted total feed consumption, total weight, grade, selling price, total revenue, total feed costs, and net revenue for good-to-choice feeder steers, weighing 850 pounds at the outset, for eight selected soilage-corn non-stilbestrol rations fed for 120 days (equation 59)[a]

Feed consumption[b] (lbs.)		Ration: All soilage		20:1		15:1		10:1	
Soilage		12,960		11,138		10,621		9,726	
Corn		--		557		708		973	
Supplement[c]		24		24		24		24	
Cost of feeder steer[d]		$212.50		$212.50		$212.50		$212.50	
Final weight		1,029		1,072		1,082		1,101	
Grade[e]		Av. Standard		High Standard		High Standard		Low Good	
Selling price		$22.38		$23.08		$23.28		$23.63	
Total revenue		$230.24		$247.33		$251.99		$260.06	
Price of soilage ($/ton)	Price of corn ($/bu.)	Total feed cost[f] ($)	Net revenue ($)	Total feed cost ($)	Net revenue ($)	Total feed cost ($)	Net revenue ($)	Total feed cost ($)	Net revenue ($)
1.00	--	7.08	10.66						
	.75			13.63	21.21	15.39	24.10	18.49	29.07
	1.00			16.11	18.72	18.55	20.94	22.83	24.72
	1.25			18.60	16.23	21.71	17.77	27.17	20.38
	1.50			21.04	13.75	24.88	14.61	31.51	16.04
	1.75			23.57	11.26	28.04	11.45	35.86	11.70
2.00	--	13.56	4.18						
	.75			19.20	15.64	20.70	18.79	23.35	24.20
	1.00			21.68	13.15	23.86	15.62	27.69	19.86
	1.25			24.17	10.67	27.03	12.46	32.04	15.52
	1.50			26.65	8.18	30.19	9.30	36.38	11.18
	1.75			29.14	5.69	33.35	6.14	40.72	68.35
3.00	--	20.04	-2.30						
	.75			24.76	10.07	26.01	13.48	28.21	19.34
	1.00			27.25	7.58	29.17	10.31	32.56	15.00
	1.25			29.74	5.00	32.34	7.15	36.90	10.66
	1.50			32.22	2.61	35.50	3.99	41.24	6.31
	1.75			34.71	.12	38.66	.83	45.58	1.97
4.00	--	26.52	-8.78						
	.75			30.33	4.50	31.32	8.17	33.08	14.48
	1.00			32.82	2.01	34.49	5.00	37.42	10.14

	1.00			32.82	2.01	34.49	5.00	37.42	10.14
	1.25			35.31	-.47	37.65	1.84	41.76	5.79
	1.50			37.79	-2.96	40.81	-1.32	46.10	1.45
	1.75			40.28	-5.44	43.97	-4.48	50.45	-2.89
5.00	--	33.00	-15.26						
	.75			35.90	-1.07	36.63	2.85	37.94	9.61
	1.00			38.39	-3.55	39.80	-.31	42.28	5.27
	1.25			40.87	-6.04	42.96	-3.47	46.62	.93
	1.50			43.36	-8.53	46.12	-6.63	50.97	-3.41
	1.75			45.85	-11.01	49.28	-9.79	55.31	-7.75
6.00	--	39.48	-21.74						
	.75			41.47	-6.64	41.94	-2.46	42.80	4.75
	1.00			43.96	-9.12	45.11	-5.62	47.15	.41
	1.25			46.44	-11.61	48.27	-8.78	51.49	-3.93
	1.50			48.93	-14.10	51.43	-11.94	55.83	-8.27
	1.75			51.42	-16.58	54.59	-15.10	60.17	-12.62
7.00	--	45.96	-28.22						
	.75			47.04	-12.21	47.26	-7.77	47.67	-.11
	1.00			49.53	-14.69	50.42	-10.93	52.01	-4.45
	1.25			52.01	-17.18	53.58	-14.09	56.35	-8.80
	1.50			54.50	-19.66	56.74	-17.25	60.69	-13.14
	1.75			56.98	-22.15	59.90	-20.41	65.03	-17.48
8.00	--	52.44	-34.70						
	.75			52.61	-17.78	52.57	-13.08	52.53	-4.97
	1.00			55.10	-20.26	55.73	-16.24	56.87	-9.32
	1.25			57.58	-22.75	58.89	-19.40	61.21	-13.66
	1.50			60.07	-25.23	62.05	-22.56	65.56	-18.00
	1.75			62.55	-27.72	65.21	-25.72	69.90	-22.34

[a]Temperature is held constant at the overall mean.

[b]The soilage and corn quantities are the same as derived in Table 35.

[c]The supplement in Table 3 is fed at the rate of .2 of a pound per day.

[d]The feeder steer is valued at $25.00/cwt.

[e]Derived from equation 46.

[f]The total feed cost includes the cost of corn and soilage plus 24 pounds of supplement valued at $2.50/cwt.

Table 52. (Continued)

Feed consumption (lbs.)		Ration: 8:1		5:1		3:1		2:1	
Soilage		9,153		7,792		6,134		4,929	
Corn		1,144		1,558		2,061		2,464	
Supplement		24		24		24		24	
Cost of feeder steer		$212.50		$212.50		$212.50		$212.50	
Final weight		1,112		1,138		1,168		1,189	
Grade[e]		Low Good		Av. Good		High Good		High Good	
Selling price		$23.85		$24.38		$25.00		$25.49	
Total revenue		$265.22		$277.50		$291.94		$303.12	
Price of soilage ($/ton)	Price of corn ($/bu.)	Total feed cost ($)	Net revenue ($)	Total feed cost ($)	Net revenue ($)	Total feed cost ($)	Net revenue ($)	Total feed cost ($)	Net revenue ($)
1.00	--								
	.75	20.50	32.22	25.37	39.63	31.30	48.14	36.07	54.55
	1.00	25.61	27.12	32.32	32.67	40.50	38.94	47.07	43.55
	1.25	30.71	22.01	39.28	25.72	49.70	29.74	58.07	32.55
	1.50	35.82	16.90	46.24	18.76	58.90	20.54	69.07	21.55
	1.75	40.93	11.79	53.20	11.80	68.10	11.34	80.07	10.54
2.00	--								
	.75	25.08	27.65	29.26	35.73	34.39	45.05	38.53	52.09
	1.00	30.18	22.54	36.22	28.78	43.59	35.85	49.53	41.08
	1.25	35.29	17.43	43.18	21.82	52.79	26.65	60.54	30.08
	1.50	40.40	12.33	50.13	14.86	61.99	17.45	71.54	19.08
	1.75	45.51	7.22	57.09	7.91	71.20	8.24	82.54	8.08
3.00	--								
	.75	29.65	23.07	33.16	31.84	37.48	41.96	41.00	49.62
	1.00	34.76	17.96	40.12	24.88	46.68	32.76	52.00	38.62
	1.25	39.87	12.86	47.07	17.92	55.88	23.56	63.00	27.62
	1.50	44.98	7.75	54.03	10.97	65.09	14.35	74.00	16.62
	1.75	50.08	2.64	60.99	4.01	74.29	5.15	85.00	5.62
4.00	--								

	1.75	50.08	2.64	60.99	4.01	74.29	5.15	85.00	5.62
4.00	--								
	.75	34.23	18.50	37.06	27.94	40.57	38.87	43.46	47.16
	1.00	39.34	13.39	44.01	20.98	49.77	29.67	54.46	36.16
	1.25	44.44	8.28	50.97	14.03	58.98	20.46	65.46	25.15
	1.50	49.55	3.17	57.93	7.07	68.18	11.26	76.47	14.15
	1.75	54.66	-1.93	64.88	.11	77.38	2.06	87.47	3.15
5.00	--								
	.75	38.81	13.92	40.95	24.05	43.66	35.78	45.93	44.69
	1.00	43.91	8.81	47.91	17.09	52.87	26.57	56.93	33.69
	1.25	49.02	3.70	54.87	10.13	62.07	17.37	67.93	22.69
	1.50	54.13	-1.40	61.82	3.17	71.27	8.17	78.93	11.69
	1.75	59.24	-6.51	68.78	-3.78	80.47	-1.03	89.93	6.87
6.00	--								
	.75	43.38	9.34	44.85	20.15	46.76	32.68	48.39	42.23
	1.00	48.49	4.24	51.80	13.19	55.96	23.48	59.39	31.23
	1.25	53.60	-.87	58.76	6.24	65.16	14.28	70.39	20.23
	1.50	58.70	-5.98	65.72	-.72	74.36	5.08	81.39	9.22
	1.75	63.81	-11.09	72.68	-7.68	83.56	-4.12	92.40	-1.78
7.00	--								
	.75	47.96	4.77	48.74	16.25	49.85	29.59	50.85	39.76
	1.00	53.07	-.34	55.70	9.30	59.05	20.39	61.86	28.76
	1.25	58.17	-5.45	62.66	2.34	68.25	11.19	72.86	17.76
	1.50	63.28	-10.56	69.61	-4.62	77.45	1.99	83.86	6.76
	1.75	68.39	-15.66	76.57	-11.57	86.66	-7.21	94.86	-4.24
8.00	--								
	.75	52.53	.19	52.64	12.35	52.94	26.50	53.32	37.30
	1.00	57.64	-4.92	59.60	5.40	62.14	17.30	64.32	26.30
	1.25	62.75	-10.02	66.55	-1.56	71.34	8.10	75.32	15.30
	1.50	67.86	-15.13	73.51	-8.51	80.55	-1.10	86.32	4.29
	1.75	72.97	-20.24	80.47	-15.47	89.75	-10.31	97.32	-6.71

Table 53. Predicted total feed consumption, total weight, grade, selling price, total revenue, total feed costs, and net revenue for good-to-choice feeder steers, weighing 850 pounds at the outset, for eight selected soilage-corn non-stilbestrol rations fed for 90 days (equation 59)[a]

Feed consumption[b] (lbs.)		Ration: All soilage		20:1		15:1		10:1	
Soilage		9,605		8,257		7,873		7,207	
Corn		--		413		525		721	
Supplement[c]		18		18		18		18	
Cost of feeder steer[d]		$212.50		$212.50		$212.50		$212.50	
Final weight		1,001		1,033		1,041		1,055	
Grade[e]		Av. Standard		High Standard		High Standard		High Standard	
Selling price		$22.90		$23.31		$23.42		$23.63	
Total revenue		$229.27		$240.81		$243.92		$249.31	
Price of soilage ($/ton)	Price of corn ($/bu.)	Total feed cost[f] ($)	Net revenue ($)	Total feed cost ($)	Net revenue ($)	Total feed cost ($)	Net revenue ($)	Total feed cost ($)	Net revenue ($)
1.00	--	5.25	11.51						
	.75			10.11	18.21	11.42	20.01	13.71	23.11
	1.00			11.95	16.36	13.76	17.66	16.92	19.89
	1.25			13.79	14.52	16.10	15.32	20.14	16.67
	1.50			15.64	12.68	18.45	12.98	23.36	13.46
	1.75			17.48	10.83	20.79	10.63	26.57	10.24
2.00	--	10.06	6.71						
	.75			14.24	14.08	15.35	16.07	17.31	19.50
	1.00			16.08	12.23	17.70	13.73	20.53	16.29
	1.25			17.92	10.39	20.04	11.38	23.74	13.07
	1.50			19.77	8.55	22.38	9.04	26.96	9.85
	1.75			21.61	6.70	24.73	6.70	30.18	6.64
3.00	--	14.86	1.91						
	.75			18.37	9.95	19.29	12.13	20.91	15.90
	1.00			20.21	8.11	21.63	9.79	24.13	12.68
	1.25			22.05	6.26	23.98	7.45	27.35	9.47
	1.50			23.89	4.42	26.32	5.10	30.56	6.25
	1.75			25.74	2.58	28.66	2.76	33.78	3.03
4.00	--	19.66	-2.89						
	.75			22.49	5.82	23.23	8.20	24.51	12.30
	1.00			24.34	3.98	25.57	5.85	27.73	[illegible]

4.00	--	19.66	-2.89						
	.75			22.49	5.82	23.23	8.20	24.51	12.30
	1.00			24.34	3.98	25.57	5.85	27.73	9.08
	1.25			26.18	2.13	27.91	3.51	30.95	5.86
	1.50			28.02	.29	30.25	1.17	34.17	2.65
	1.75			29.87	-1.55	32.60	-1.18	37.38	-.57
5.00	--	24.46	-7.70						
	.75			26.62	1.69	27.16	4.26	28.12	8.69
	1.00			28.47	-.15	29.51	1.92	31.34	5.48
	1.25			30.31	-1.99	31.85	-.43	34.55	2.26
	1.50			32.15	-3.84	34.19	-2.77	37.77	-.96
	1.75			34.00	-5.68	36.53	-5.11	40.99	-4.17
6.00	--	29.26	-12.50						
	.75			30.75	-2.44	31.10	.32	31.72	5.09
	1.00			32.59	-4.28	33.44	-2.02	34.94	1.87
	1.25			34.44	-6.12	35.78	-4.36	38.16	-1.34
	1.50			36.28	-7.97	38.13	-6.70	41.37	-4.56
	1.75			38.12	-9.81	40.47	-9.05	44.59	-7.78
7.00	--	34.07	-17.30						
	.75			34.88	-6.57	35.03	-3.61	35.32	1.49
	1.00			36.72	-8.41	37.38	-5.96	38.54	-1.73
	1.25			38.57	-10.25	39.72	-8.30	41.76	-4.95
	1.50			40.41	-12.10	42.06	-10.64	44.98	-8.16
	1.75			42.25	-13.94	44.41	-12.98	48.19	-11.38
8.00	--	38.87	-22.10						
	.75			39.01	-10.69	38.97	-7.55	38.93	-2.12
	1.00			40.85	-12.54	41.31	-9.89	42.15	-5.33
	1.25			42.70	-14.38	43.66	-12.23	45.36	-8.55
	1.50			44.54	-16.22	46.00	-14.58	48.58	-11.77
	1.75			46.38	-18.07	48.34	-16.92	51.80	-14.98

[a]Temperature is held constant at the overall mean.

[b]The soilage and corn quantities are the same as derived in Table 35.

[c]The supplement in Table 3 is fed at the rate of .2 of a pound per day.

[d]The feeder steer is valued at $25.00/cwt.

[e]Derived from equation 46.

[f]The total feed cost includes the cost of corn and soilage plus 18 pounds of supplement valued at $2.50/cwt.

Table 53. (Continued)

Feed consumption (lbs.)		Ration: 8:1		Ration: 5:1		Ration: 3:1		Ration: 2:1	
Soilage		6,779		5,763		4,561		3,625	
Corn		847		1,153		1,520		1,812	
Supplement		18		18		18		18	
Cost of feeder steer		$212.50		$212.50		$212.50		$212.50	
Final weight		1,064		1,085		1,108		1,126	
Grade		High Standard		Low Good		Av. Good		Av. Good	
Selling price		$23.76		$24.06		$24.43		$24.71	
Total revenue		$252.77		$260.98		$270.67		$278.17	
Price of soilage ($/ton)	Price of corn ($/bu.)	Total feed cost ($)	Net revenue ($)	Total feed cost ($)	Net revenue ($)	Total feed cost ($)	Net revenue ($)	Total feed cost ($)	Net revenue ($)
1.00	--								
	.75	15.19	25.08	18.77	29.72	23.09	35.08	26.53	39.14
	1.00	18.97	21.30	23.91	24.57	29.88	28.29	34.62	31.05
	1.25	22.75	17.51	29.06	19.43	36.66	21.50	42.72	22.96
	1.50	26.54	13.73	34.20	14.28	43.45	14.71	50.81	14.87
	1.75	30.32	9.95	39.35	9.14	50.24	7.93	58.90	6.78
2.00	--								
	.75	18.58	21.69	21.65	26.84	25.37	32.80	28.35	37.33
	1.00	22.36	17.91	26.79	21.69	32.16	26.01	36.44	29.24
	1.25	26.14	14.13	31.94	16.55	38.95	19.22	44.53	21.15
	1.50	29.93	10.34	37.08	11.40	45.73	12.43	52.62	13.06
	1.75	33.71	6.56	42.23	6.25	52.52	5.65	60.71	4.97
3.00	--								
	.75	21.97	18.30	24.53	23.95	27.65	30.51	30.16	35.52
	1.00	25.75	14.52	29.67	18.81	34.44	23.73	38.25	27.42
	1.25	29.53	10.74	34.82	13.66	41.23	16.94	46.34	19.33
	1.50	33.32	6.95	39.96	8.52	48.01	10.15	54.43	11.24
	1.75	37.10	3.17	45.11	3.37	54.80	3.37	62.52	3.15
4.00	--								
	.75	25.36	14.91	27.41	21.07	29.93	28.23	31.97	33.70
	1.00	29.14	11.13	32.56	15.93	36.72	21.45	40.06	25.61
	1.25	32.92	7.35	37.70	10.78	43.51	14.66	48.15	17.52
	1.50	36.71	3.56	42.85	5.64	50.29	7.87	56.24	9.43
	1.75	40.49	-.22	47.99	.49	57.08	1.09	64.33	1.34

	.75	25.36	14.91	27.41	21.07	29.93	28.23	31.97	33.70
	1.00	29.14	11.13	32.56	15.93	36.72	21.45	40.06	25.61
	1.25	32.92	7.35	37.70	10.78	43.51	14.66	48.15	17.52
	1.50	36.71	3.56	42.85	5.64	50.29	7.87	56.24	9.43
	1.75	40.49	-.22	47.99	.49	57.08	1.09	64.33	1.34
5.00	--								
	.75	28.75	11.52	30.29	18.19	32.21	25.95	33.78	31.89
	1.00	32.53	7.74	35.44	13.05	39.00	19.17	41.87	23.80
	1.25	36.31	3.96	40.58	7.90	45.79	12.38	49.96	15.71
	1.50	40.10	.17	45.73	2.76	52.57	5.59	58.06	7.62
	1.75	43.88	-3.61	50.87	-2.39	59.36	-1.19	66.15	-.47
6.00	--								
	.75	32.14	8.13	33.17	15.31	34.49	23.67	35.60	30.08
	1.00	35.92	4.35	38.32	10.17	41.28	16.89	43.69	21.99
	1.25	39.70	.57	43.46	5.02	48.07	10.10	51.78	13.90
	1.50	43.49	-3.22	48.61	-.13	54.85	3.31	59.87	5.81
	1.75	47.27	-7.00	53.75	-5.27	61.64	-3.47	67.96	-2.28
7.00	--								
	.75	35.53	4.74	36.05	12.43	36.77	21.39	37.41	28.27
	1.00	39.31	.96	41.20	7.28	43.56	14.61	45.50	20.18
	1.25	43.09	-2.82	46.35	2.14	50.35	7.82	53.59	12.08
	1.50	46.88	-6.61	51.49	-3.01	57.13	1.03	61.68	3.99
	1.75	50.66	-10.39	56.64	-8.15	63.92	-5.75	69.77	-4.10
8.00	--								
	.75	38.92	1.35	38.94	9.55	39.05	19.11	39.22	26.45
	1.00	42.70	-2.43	44.08	4.40	45.84	12.33	47.31	18.36
	1.25	46.48	-6.21	49.23	-.74	52.63	5.54	55.40	10.27
	1.50	50.26	-10.00	54.37	-5.89	59.41	-1.25	63.49	2.18
	1.75	54.05	-13.78	59.52	-11.03	66.20	-8.03	71.58	-5.91

total weight equations and b_i's ($i = 1, \ldots, 5$) refer to the constants in the price equations. Since there exists a certain relationship between feed inputs and time as specified by the soilage consumption functions, then in order to maximize profits they must be maximized subject to the conditions specified by the soilage consumption functions. With this restriction the profit function, equation 60, can be written as:

$$(61)\quad \pi = (a_1 + a_2C + a_3F + a_4C^2 + a_5F^2 + a_6CF + a_7H)\,(b_1 + b_2F + b_3T + b_4T^2 + b_5FT) - P_CC - P_FF - .2TP_S - K - \lambda\,(F - c_1C - c_2T - c_3C^2 - c_4T^2 - c_5CT - c_5H)$$

where λ is an undetermined Lagrange multiplier and the c_i's ($i = 1, \ldots, 5$) are the constants in the soilage consumption functions.

If the ration equation is defined as:

$$(62)\quad \frac{C}{F} = \omega$$

then

$$(63)\quad C = \omega F.$$

Now by substituting ωF for C in the profit function, equation 61, it will be possible to determine the optimum feeding period for any given ration and the quantities of corn and soilage that will be fed during this optimum feeding period. Thus, the profit function, equation 61, can now be written as:

$$(64)\quad \pi = \Big[(a_4\omega^2 + a_5 + a_6\omega)F^2 + (a_2\omega + a_3)F + a_1 + a_7H\Big]\Big[b_1 + b_2F + b_3T + b_4T^2 + b_5FT\Big] - (P_C\omega + P_F)F - .2TP_S - K - \lambda\Big[(1 - c_1\omega)F - c_2T - c_3\omega^2F^2 - c_4T^2 - c_5\omega FT - c_6H\Big].$$

Maximization of the profit function subject to the conditions of the soilage consumption function results in the following set of necessary conditions:

$$(65)\quad \frac{\partial \pi}{\partial F} = \Big[(a_4\omega^2 + a_5 + a_6\omega)F^2 + (a_2\omega + a_3)F + a_1 + a_7H\Big]\Big[b_2 + b_5T\Big] + \Big[b_1 + b_2F + b_3T + b_4T^2 + b_5FT\Big]\Big[2(a_4\omega^2 + a_5 + a_6\omega)F + (a_2\omega + a_3)\Big] - (P_C\omega + P_F) - \Big[(1 - c_1\omega)\lambda - 2c_3\omega^2\lambda F - c_5\omega\lambda T\Big] = 0$$

$$(66)\quad \frac{\partial \pi}{\partial T} = \Big[(a_4\omega^2 + a_5 + a_6\omega)F^2 + (a_2\omega + a_3)F + a_1 + a_7H\Big]\Big[b_3 + 2b_4T + b_5F\Big] - .2P_S - \Big[-\lambda c_2 - 2c_4\lambda T^2 - c_5\omega F\lambda\Big] = 0.$$

There are now three equations (the soilage consumption function and equations 65 and 66) and three unknowns (F, T and λ) and the solution of these equations will determine the optimum feeding time and the quantity of soilage (F) that will be fed given the ration ω and the feed price combination. If corn is included in the ration, then the quantity of corn that will be fed can be determined from the ration equation 63. Once the optimum feeding period and the quantities of soilage and corn have been determined it is possible to determine also what the profits will be for this optimum feeding period by substituting the values of soilage (F), corn (C) and time (T) into the profit equation (either equation 58 or equation 59 depending on whether or not stilbestrol has been fed in the ration).

For any given soilage-corn ration and feed price combination, the optimum feeding period is limited by the pasture growing season which is approximately 140 days in length. Therefore, for any given soilage-corn ration and feed price combination, the optimum feeding period can not exceed the pasture growing season.

The optimum feeding period for the 20:1 soilage-corn stilbestrol ration with soilage valued at $6.00 per ton and corn valued at $1.00 per bushel is a feeding period of 28 days. During this feeding period of 28 days 2,139 pounds of soilage, 107 pounds of corn and 5.6 pounds of supplement would be fed. The profit at the end of the 28 day feeding period is predicted to be 28 cents which is the maximum amount of profit that may be expected from feeding the 20:1 soilage-corn ration with soilage valued at $6.00 per ton and corn valued at $1.00 per bushel.

The optimum feeding period for the 20:1 soilage-corn stilbestrol ration with different feed price assumptions could be solved in a similar manner. Moreover, the same procedure could be applied to all possible soilage-corn rations either with or without stilbestrol.

While the above procedure can be used to determine what the optimum feeding period would be for any given soilage-corn ration and feed price combination, it does not, however, specify what is the optimum soilage-corn ration. In order to determine the optimum soilage-corn ration one additional necessary condition must be added to the necessary conditions above (i.e., equations 65 and 66). This additional necessary condition is:

$$(67) \quad \frac{\partial \pi}{\partial \omega} = \left[b_1 + b_2F + b_3T + b_4T^2 + b_5FT\right] \left[2a_4\omega F^2 + a_6F^2 + a_2F\right] - P_CF - \left[- c_1\lambda F - 2c_3\omega\lambda F - c_5\lambda FT\right] = 0.$$

There are now four equations (the soilage consumption function and equations 65, 66 and 67) and four unknowns (F, T, λ and ω). The solution of these equations will determine the optimum ration, the optimum feeding time and the quantity of soilage (F) that will be fed. Once the quantity of soilage and the ration ω have been determined the quantity of corn that will be fed is readily determined from the ration equation 63.

For any given feed price combination the optimum ration is limited by the 2:1 soilage-corn ration. Rations of less than 2 parts soilage to 1 part corn are outside the limits of this study. Therefore, the optimum ration can not be less than 2 parts soilage to 1 part corn and the optimum feeding period can not exceed the pasture growing season which is approximately 140 days.

The optimum soilage-corn stilbestrol ration and the optimum feeding period with soilage valued at $6.00 per ton and corn valued at $1.00 per bushel is the 2:1 soilage-corn ration fed for the entire pasture season or 140 days. The profit is predicted to be $46.45.

The optimum soilage-corn stilbestrol ration and the optimum feeding period could also be determined for different feed price assumptions. Also, the optimum soilage-corn ration and the optimum feeding period for the non-stilbestrol soilage-corn rations under various feed price assumptions would be determined in identically the same manner as they were for the stilbestrol rations.

SUMMARY AND CONCLUSIONS

Relatively little is known of how pasture forages and corn substitute for each other in a beef fattening enterprise. Yet without such information it is difficult to determine which combination of pasture forage and corn should be fed if profits are to be maximized. Profit maximization in a beef fattening enterprise depends not only upon the cost of feed but also upon the time of marketing. The pasture forage-corn ration that minimizes costs may not necessarily be the ration that maximizes profits since profits are also affected by the time of marketing. During the beef fattening period both the quality and the price of beef are subject to change. Consequently, the beef cattle feeder is confronted with the problem of selecting the least-cost pasture forage-corn ration that will place the beef cattle on the market finished to a grade at the time when the expected market price is such as to maximize profits.

A beef feeding experiment was specially designed to determine the feed relationships of soilage (fresh-chopped pasture forage) and corn. The experiment was conducted at two locations over a period of three years, 1957, 1958 and 1959. Six different soilage-corn rations from all soilage to 2 parts soilage and 1 part corn were fed to different lots of feeder steers at each location. The rations at each location were also fortified with a feed supplement. At one of the locations stilbestrol was included in the rations whereas at the other location no stilbestrol was fed. The results of this feeding experiment are based on the performance of 336 head of good-to-choice feeder steers.

Input-output relationships and substitution rates were derived from multiple regression equations for the stilbestrol rations, the non-stilbestrol rations and the stilbestrol and non-stilbestrol rations combined. Several alternative regression equations were used in this study, including quadratic, modified Cobb-Douglas and exponential functions. In each of the above functions an attempt has been made to remove the effects of autocorrelation by estimating an autocorrelation coefficient and then making an autoregressive transformation.

While the quadratic functions gave better results than either the modified Cobb-Douglas or the exponential, more research is needed in order to determine which functions are the best under different situations. In some cases the modified Cobb-Douglas function gave good results, while in other cases, the function gave increasing returns to scale denoting that a small proportional increase in the quantity of feed fed results in a more than proportionate increase in beef gain. These results are inconsistent with nutrition and production theory. The exponential functions, which merit further research, gave sigmoid isoquant contours denoting first increasing marginal rates of substitution between feeds and then decreasing marginal rates of substitution. Again these results are inconsistent with nutrition and production logic.

The quadratic production functions for rations with stilbestrol, without stilbestrol and the aggregate function with the stilbestrol and non-stilbestrol rations combined are:

I. With stilbestrol

$$G = .11637150C + .02316051F - .0000049955C^2$$
$$- .0000007455F^2 + .0000000374CF - 1.2236046H$$

II. Without stilbestrol

$$G = .149718120 + .02128774F - .0001226120C^2$$
$$- .0000007455F^2 - .0000037907CF - 2.2005042H$$

III. The aggregate function

$$G = .13628727C + .02193828F - .000008190C^2$$
$$- .00000063F^2 - .00000253CF - 1.75011550H.$$

In these equations G refers to pounds of beef gain, C refers to pounds of corn, F refers to pounds of soilage and H refers to the deviations of the average maximum temperature of each observation interval from the mean maximum temperature for the "overall" feeding period. From the above production functions, the basic input-output relationships can be derived.

The marginal rates of substitution of corn for soilage have been derived for various soilage-corn rations at various levels of beef gain. The marginal rates of substitution indicate, for a given level of gain, the pounds of soilage which could be replaced if an additional pound of corn were added to the ration. For 100 pounds of beef gain, the marginal rate of substitution of corn for soilage is 6.57 for the stilbestrol 20:1 soilage-corn ration and 5.17 for the 2:1 ration. For the same level of gain, the marginal rate of substitution of corn for soilage is 8.11 for the non-stilbestrol 20:1 soilage-corn ration and 7.36 for the 2:1 ration. The marginal rates of substitution of corn for soilage are at a diminishing rate. Similar results were obtained for higher levels of beef gains.

For a given level of gain, the least-cost ration is specified where the marginal rate of substitution is equal to the inverse feed price ratio, *i.e.*,

$$\frac{\partial F}{\partial C} = \frac{\text{price per pound of corn}}{\text{price per pound of soilage}} .$$

Least-cost rations for various levels of gain with various corn-soilage price ratios have been determined for both the stilbestrol and non-stilbestrol rations. The least-cost ration in terms of gain may not necessarily be the ration that results in the maximum profits since different rations produce different grades (quality) of beef which sell for different prices. Furthermore, as cattle feeders well know, the time at which cattle are marketed may affect profits as much or more than the cost of feed.

Time equations for the stilbestrol and non-stilbestrol rations, which were derived from estimated soilage consumption functions, express the total time (T) required to consume a given quantity of corn and soilage as a function of the soilage (F) and corn (C) fed. The time equations for the stilbestrol and non-stilbestrol rations are:

I. With stilbestrol

$$T = -558.36128626 + .05781948C$$
$$\pm 6.7475645\,(6{,}847.57044400 - .00000523C^2$$
$$- .82767919C - .26107315H + .29640324F)^{1/2}$$

II. Without stilbestrol

$$T = -1{,}176.48647060 + .00763899C$$
$$\pm 11.436840\,(10{,}582.2245560 - .00001571C^2$$
$$+ .45460922C - 3.07787452H + .17108144F)^{1/2}.$$

The time equations can be used to predict the time required to produce various levels of gain, for different soilage-corn rations, by substituting into the time equations the predicted feed requirements for the various levels of gain. The time required to produce a given level of gain, for both the stilbestrol and non-stilbestrol rations, decreases as the proportion of corn in the ration increases. Also, for a given feeding period the maximum level of gain is attained with the heaviest corn ration (i.e., the 2:1 soilage-corn ration).

The beef steers were also graded at definite intervals during the feeding period. After the grade observations (which were in subjective grade terms) had been coded, a functional relationship that expresses grade as a function of the corn and soilage fed was estimated for both the stilbestrol and non-stilbestrol rations. The overall equations for estimating the grade of beef steers fed various soilage-corn rations either with or without stilbestrol are:

I. With stilbestrol

$$Q = 21.67 + .0024655079C + .0000220680F - .0000003510C^2$$

II. Without stilbestrol

$$Q = 21.67 + .0016294178C + .0000270836F - .0000000330C^2$$

where Q is the predicted slaughter grade which can be interpreted in subjective grade terms in Table 39. From these grade functions, the iso-grade equations as well as the marginal rates of substitution equations can be derived.

In order to estimate the profits from feeding different soilage-corn rations for different length feeding periods with different feed price assumptions, it was necessary to derive a price function that would estimate the price of the beef steers during the feeding period. These price functions must represent the grade of the beef steers during the feeding period as well as the market price associated with the grade. The estimated price functions were used in the overall stilbestrol and non-stilbestrol profit functions. The profit equations are:

I. With stilbestrol

$$\pi = (850.00 + .11637150C + .02316051F - .0000049955C^2 - .0000007455F^2 + .0000000374CF - 1.2236046H)(.2500 - .0000040158F + .0000382807T + .0000017537T^2 - .0000000048FT) - P_C C - P_F F - .2TP_S - K$$

II. Without stilbestrol

$$\pi = (850.00 + .14971812C + .02128774F - .0000122612C^2 - .0000005775F^2 - .0000037907CF - 2.2005042H)(.2500 - .0000004996F - .0002393978T + .0000036576T^2 - .0000000281FT) - P_C C - P_F F - .2TP_S - K$$

where π refers to the profit, P_C refers to the price of corn, P_F refers to the price of soilage, P_S refers to the price of the supplement, K is the value of the feeder steer at the beginning of the feeding period and all of the other symbols are the same as explained earlier.

Estimated profits from feeding beef steers various soilage-corn rations for various length feeding periods under various feed price assumptions have been tabulated. Generally, the greatest profits are obtained by feeding the heaviest corn ration. However, when the price of soilage is low relative to the price of corn, then the most profitable ration is a ration of less corn and more soilage.

The equations and the procedure is also given for obtaining the optimum feeding period for any given soilage-corn ration under different feed price assumptions. Similarly, the equations and the procedure is given for obtaining the optimum soilage-corn ration with different feed price assumptions.

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APPENDIX A

The Exponential and Modified Cobb-Douglas Production Functions*

In addition to the single equation quadratic model discussed in the text, two other models were investigated in an attempt to estimate the beef cattle production function. The first model is an exponential model involving a system of equations and the second model is a modified Cobb-Douglas function.

The exponential function

The first model to be presented is the exponential model involving a system of equations. This system of equations have a special form in that they form a recursive system (15, pp. 64-65). The recursive system of equations consists of the following relations:

(68) The production function

$$\log G_t + c_1 \log F_t = c_2 + c_3 R_t + c_4 R_t^2 + c_5 H_t + u_{3t}$$

(69) The ration relation

$$C_t / F_t = R_t$$

(70) The gain relation

$$\log G_t = a_1 + a_2 \log T_t + a_3 R_t + a_4 R_t^2 + a_5 H_t + u_{1t}$$

---

*The author is indebted to Dr. Wayne A. Fuller for suggesting the functional form of the two functions discussed in this section.

(71) The soilage consumption relation

$$\log F_t = b_1 + b_2 \log T_t + b_3 R_t + b_4 R_t^2 + b_5 H_t + u_{2t}$$

where

(72) $u_{3t} = u_{1t} + c_1 u_{2t}$

(73) $u_{30} = u_{10} = u_{20} = 0$

and G refers to pounds of beef gains, F refers to pounds of soilage, C refers to pounds of corn, R refers to the ratio of corn to soilage (ration), T refers to time in days, H refers to temperature,* the a_i's and b_i's ($i = 1, \ldots, 5$) are constants to be estimated, the u_j's ($j = 1, 2$) are random variables and t is an index of time denoting the observation period. The c_i's ($i = 1, \ldots, 5$) in equation 68 are known functions of the constants to be estimated in equations 70 and 71, that is,

(74)

(a) $c_1 = -\frac{a_2}{b_2}$

(b) $c_2 = a_1 - \frac{a_2}{b_2} b_1$

(c) $c_3 = a_3 - \frac{a_2}{b_2} b_3.$

(d) $c_4 = a_4 - \frac{a_2}{b_2} b_4$

(e) $c_5 = a_5 - \frac{a_2}{b_2} b_5$

---

*The temperature observation is the average maximum temperature for the observation period. All of the other variables are measured in the same manner as discussed in an earlier section (see page 56 ff.).

Equations 70 and 71 are estimated by the method of least squares and then the two equations are combined to give equation 68. Equation 69 is an identification equation and does not need to be estimated.

The model consists of two endogenous variables G and F, and four exogenous variables T, R, R^2, and H. The reasoning behind these relations is that both the beef gains (G) and the soilage consumption (F) are experimentally determined whereas time (T) and the ration (R) and ration squared (R^2) are predetermined variables while temperature is truly an exogenous variable.

In order to consider autocorrelation, as with the quadratic function, the assumption was made that the random variables, u_j's ($j = 1, 2$), were generated by the autoregressive schemes:

(75) $u_{1t} = \beta_1 u_{1t-1} + e_{1t}$

(76) $u_{2t} = \beta_2 u_{2t-1} + e_{2t}$

where

(a) the errors, e_{it}, are uncorrelated with each of the independent variables in each equation.

(77) (b) $E(e_{it}) = 0$ and the e_{it}'s are normally distributed

(c) $E(e_{it}e_{jt}) = \sigma_{ij} < \infty$

(d) $E(e_{it}e_{js}) = 0$; $i = 1, 2$; $j = 1, 2$; $t \neq s$

and the β_i's are the autocorrelation coefficients.

Under the assumptions in equation 75, equation 70 may be written as:

$$(78)\quad (\log G_t - \beta_1 \log G_{t-1}) = \log a_1(1 - \beta_1) + a_2(\log T_t - \beta_1 \log T_{t-1}) + a_3(R_t - \beta_1 R_{t-1}) + a_4(R_t^2 - \beta_1 R_{t-1}^2) + a_5(H_t - \beta_1 H_{t-1}) + e_{1t}.$$

Similarly, under the assumptions of equation 76, equation 71 may be written as:

$$(79)\quad (\log F_t - \beta_2 \log F_{t-1}) = \log b_1(1 - \beta_2) + b_2(\log T_t - \beta_2 \log T_{t-1}) + b_3(R_t - \beta_2 R_{t-1}) + b_4(R_t^2 - \beta_2 R_{t-1}^2) + b_5(H_t - \beta_2 H_{t-1}) + e_{2t}.$$

If the variables in equations 70 and 71 are replaced by the transformed variables in equations 78 and 79, respectively, then the error terms are not autocorrelated. An empirical estimate of the autocorrelation coefficient, β_1, in equation 75, was made in a manner similar to the procedure discussed in the text (see page 64). The autocorrelation coefficient β_1 estimated by this procedure was .57596153 with a standard error of .07509728. This coefficient was highly significant at the .001 per cent level of confidence.*

The autocorrelation coefficient β_1 was also used as an estimate of the autocorrelation coefficient β_2 in equation

*The "t" value for the estimated coefficient is 7.6695 with 143 degrees of freedom.

76. The reasons for using the same estimate of the autocorrelation coefficient in both equations 78 and 79 are the same reasons discussed earlier in the text (see page 106).

When the original data were transformed to logarithms the variances between the time periods (i.e., the observation periods) were no longer homogeneous. Since the variance for the first time period (i.e., the first observation period) was approximately four times the variance of the other time periods, the first observations were weighted by dividing all the variables for the first observation period by two. This procedure tended to restore the homogeneity of the variance between time periods.

The estimated equations The estimated gain functions for the overall stilbestrol and non-stilbestrol rations are:

I. With stilbestrol

$$(80)\quad \log G = .89782288 + .72323783 \log T + 1.47167010R - 1.91775510R^2 - .00236429H$$

II. Without stilbestrol

$$(81)\quad \log G = 1.06433880 + .64511669 \log T + 1.2632370R - 1.69572540R^2 - .00188145H.$$

The estimated soilage consumption functions for the overall stilbestrol and non-stilbestrol rations are:

I. With stilbestrol

$$(82)\quad \log F = -.11743472 + 1.04132470 \log T - 1.03246990R + .54436501R^2 - .00002339H$$

II. Without stilbestrol

(83) $\log F = -.20054633 + 1.08377530 \log T$
$- 1.24860120R + .75196945R^2 + .00073851H.$

The production functions for the overall stilbestrol and non-stilbestrol rations are:

I. With stilbestrol

(84) $G = .389326\ F^{.6945363}\ e^{5.0398013R-5.2863577R^2-.0054066H}$

II. Without stilbestrol

(85) $G = .623860\ F^{.6440521}\ e^{4.8521485R-5.1475925R^2-.0054288H}.$

The coefficient of determination, standard errors and the "t" values for the overall gain and soilage consumption functions are presented in Tables 54 and 55 for the stilbestrol rations and in Tables 56 and 57 for the non-stilbestrol rations, respectively. The variances of the coefficients in the production functions, as shown in equation 74, have not been computed. However, the variances of these coefficients, which are only approximate, may be computed. The variance-covariance matrix for these coefficients can be easily shown by making use of matrix algebra.

Let

(86) $[XX']^{-1}$ = The inverse matrix of the sum of squares and cross-products.

Table 54. Coefficient of determination, standard errors and "t" values for the overall stilbestrol gain function (equation 80)

R^2	Independent variable	Standard error of regression coefficient	"t" value	Level of significance
.9788	(constant)	.06226914	14.418	$p < .001$
	log T	.02165814	33.393	$p < .001$
	R	.16934600	8.690	$p < .001$
	R^2	.42343700	4.529	$p < .001$
	H	.00062402	3.789	$p < .001$

Table 55. Coefficient of determination, standard errors and "t" values for the overall stilbestrol soilage consumption function (equation 82)

R^2	Independent variable	Standard error of regression coefficient	"t" value	Level of significance
.9989	(constant)	.01670231	7.031	$p < .001$
	log T	.00580931	179.251	$p < .001$
	R	.04542330	22.730	$p < .001$
	R^2	.11357800	4.793	$p < .001$
	H	.00016733	.140	$p > .50$

Table 56. Coefficient of determination, standard errors and "t" values for the overall non-stilbestrol gain function (equation 81)

R^2	Independent variable	Standard error of regression coefficient	"t" value	Level of significance
.9675	(constant)	.06564631	16.213	$p < .001$
	log T	.02309004	27.939	$p < .001$
	R	.18298530	6.904	$p < .001$
	R^2	.45061000	3.763	$p < .001$
	H	.00061368	3.066	$.001 < p < .005$

Table 57. Coefficient of determination, standard errors and "t" values for the overall non-stilbestrol soilage consumption function (equation 83)

R^2	Independent variable	Standard error of regression coefficient	"t" value	Level of significance
.9980	(constant)	.02320912	8.641	$p < .001$
	log T	.00816344	132.760	$p < .001$
	R	.06469410	19.300	$p < .001$
	R^2	.15931100	4.720	$p < .001$
	H	.00021700	3.403	$p < .001$

$$(87)\quad \begin{bmatrix} \frac{\partial c_1}{\partial a_1} & \cdots & \frac{\partial c_1}{\partial a_5} & \frac{\partial c_1}{\partial b_1} & \cdots & \frac{\partial c_1}{\partial b_5} \\ \vdots & & \vdots & \vdots & & \vdots \\ \frac{\partial c_5}{\partial a_1} & \cdots & \frac{\partial c_5}{\partial a_5} & \frac{\partial c_5}{\partial b_1} & \cdots & \frac{\partial c_5}{\partial b_5} \end{bmatrix} = \theta = \text{A matrix of the partial derivatives of the production function coefficients}$$

$$(88)\quad \begin{bmatrix} [XX']^{-1}\ \sigma_{11} & [XX']^{-1}\ \sigma_{12} \\ [XX']^{-1}\ \sigma_{12} & [XX']^{-1}\ \sigma_{22} \end{bmatrix} = \Delta = \text{The variance matrix.}$$

The variance-covariance matrix for the production function coefficients may now be formed by

$$(89)\quad \theta \Delta \theta' = \phi = \text{The variance-covariance matrix for the production function coefficients.}$$

Even though the coefficients of determination for this model of the beef cattle production function were quite high, the model had to be rejected on the basis of nutrition and production logic. The beef gain isoquants derived from this model were sigmoid curves denoting first increasing marginal rates of substitution between feeds and then decreasing marginal rates of substitution.

While this particular model did not give reasonable results based on nutrition and production logic, nevertheless, the model merits further research.

The modified Cobb-Douglas function

The second model that was investigated in an attempt to estimate the beef cattle production function was a modified Cobb-Douglas function. While the Cobb-Douglas function possesses certain important advantages, such as ease of estimation, the function also possesses certain shortcomings which are inherent in the mathematical form of the function.

Some of the characteristic shortcomings of the Cobb-Douglas function are: 1) constant elasticity of production; 2) the isoquant curves are asymptotic to the axes and hence will never intersect the axes, indicating that the product output will be zero whenever one of the resource inputs is zero; 3) along any given scale line, the slope of successive isoquants is the same; and 4) the function only allows for either increasing, constant or decreasing marginal returns.

In order to use the Cobb-Douglas function to estimate the beef cattle production function it is necessary to modify the function to overcome the shortcomings of symmetry. In feeding beef cattle, various rations from all forage to various combinations of corn and forage may be fed. Thus for the all forage ration the corn input is zero. If beef gains are to be estimated with the classical Cobb-Douglas function, where beef gains = g(corn, forage), then beef gains will be zero whenever the cattle are fed the all forage ration. However, the function can be modified by replacing the feed input variable corn (C) with $(C + \alpha)$. The modified Cobb-Douglas function for estimating the beef cattle production function can then be written as

$$(90) \quad \log G_t = \log a_1 + a_2 \log F_t + a_3 \log(C + \alpha)_t + a_4 H_t + u_t$$

where G refers to pounds of beef gains, F refers to pounds of soilage, C refers to pounds of corn, H refers to temperature,* the a_i's $(i = 1, \ldots, 4)$ are constants to be estimated, u is a random variable, t is an index of time denoting the observation period and "α" is a constant that is to be estimated such that

$$(91 \quad \sum u_t^2 = \text{minimum}.$$

In order to consider autocorrelation the assumption was made that the random variable u_t was generated by the autoregressive scheme:

$$(92) \quad u_t = \beta u_{t-1} + e_t$$

where β is the autocorrelation coefficient and

(a) the errors, e_t, are uncorrelated with each of the independent variables in the equation

(93) (b) $E(e_t) = 0$ and the e_t's are normally distributed

(c) $E(e_t^2) = \sigma^2 \qquad < \infty$

(d) $E(e_t e_s) = 0 \qquad t \neq s$

(e) $e_0 = 0$

---

*All of the variables (G, F, C, and H) are measured in the same manner as with the exponential function (see page 195).

Under the assumptions of equation 92, equation 90 can now be written as:

$$(94)\quad (\log G_t - \beta \log G_{t-1}) = \log a_1(1 - \beta) + a_2(\log F_t - \beta \log F_{t-1}) + a_3\left[\log (C + \alpha)_t - \beta \log (C + \alpha)_{t-1}\right] + a_4(\log H_t - \beta \log H_{t-1}) + e_t.$$

If the variables in equation 90 are replaced with the transformed variables in equation 94, then the error terms will not be autocorrelated. The autocorrelation coefficient, β, used to transform the data was the same coefficient used to transform the data in the exponential function discussed in the previous section. Similarly, the first observations were weighted in the same manner as in the exponential function and for the same reasons (see page 198).

<u>The estimated equations</u> The estimated production functions for the overall stilbestrol and non-stilbestrol rations are:

I. With stilbestrol

$$(95)\quad G = .06413187\, F^{.45535483} (C + 400)^{.59896829}\, e^{-.00185945H}$$

II. Without stilbestrol

$$(96)\quad G = .091115840\, F^{.38061675} (C + 600)^{.62111655}\, e^{-.00155832H}.$$

The computed coefficient of determination for the overall stilbestrol production function (equation 95) is .9759 while the coefficient of determination for the overall non-stilbestrol production function (equation 96) is .9631. The approximate variances of the estimated regression coefficients and the constant α may be computed. The procedure for deriving the variance-covariance matrix for estimates of the approximate variances of the regression coefficients and the constant α can easily be shown with the aid of matrix algebra.

The production function in equations 95 or 96 can be written in the general form as:

$$(97) \quad \ln G = \ln c_1 + c_2 \ln F + c_3 \ln (C + \alpha) + c_4 H$$

where the c_i's ($i = 1, \ldots, 4$) are the estimated regression coefficients, ln denotes the natural logarithms of the production function and all of the other symbols are the same as defined in the previous section.

The partial derivatives of ln G with respect to the estimated constants gives the following variables which are used to make up the inverse matrix:

$$\begin{aligned}
&(1) \quad \frac{\partial \ln G}{\partial c_1} = \frac{1}{c_1} \\
&(2) \quad \frac{\partial \ln G}{\partial c_2} = \ln F \\
(98) \quad &(3) \quad \frac{\partial \ln G}{\partial c_3} = \ln (C + \alpha) \\
&(4) \quad \frac{\partial \ln G}{\partial \alpha} = \frac{c_3}{(C + \alpha)} \\
&(5) \quad \frac{\partial \ln G}{\partial c_4} = H.
\end{aligned}$$

Now let Z_j ($j = 1, \ldots, 5$) be a row vector of n observations of the variable so defined by the jth equation in 98

$$
(99)\quad
\begin{aligned}
&(a)\ Z_1 = \left[\frac{1}{c_1}\right]\\
&(b)\ Z_2 = [\ln F]\\
&(c)\ Z_3 = [\ln (C + \alpha)]\\
&(d)\ Z_4 = \left[\frac{c_3}{(C + \alpha)}\right]\\
&(e)\ Z_5 = [H]
\end{aligned}
$$

(*e.g.*, Z_1 is a row vector of n "$1/c_1$" values while Z_2 is a row vector of the n soilage observations in natural logarithms etc.).

Let Z be a matrix composed of the Z_j row vectors so that

$$(100)\quad [ZZ']^{-1} = \text{the inverse matrix}$$

and

$$(101)\quad [ZZ']^{-1}\, s^2 = \phi = \text{the variance-covariance matrix}$$

where

$$(102)\quad s^2 = \frac{\sum (\ln G - \ln \hat{G})^2}{n - r}$$

and r is the number of constants to be estimated. The variance of the estimated regression coefficients, the c_i's, and the constant α are now readily obtained from the variance-covariance matrix, ϕ.

The standard errors and the "t" values for the estimated constants in equations 95 and 96 have not been computed. However, the procedure for computing the approximate variances has been presented and if the model has not been rejected the variance-covariance matrix, ϕ, would have been computed.

Even though the coefficients of determination were quite high for this model of the beef cattle production function, the model was rejected on the basis of nutrition and production logic. The estimated overall stilbestrol production function gave increasing returns to scale denoting that a small proportional increase in the quantity of feed fed results in a more than a proportionate increase in beef gains. These results are inconsistent with nutrition and production theory and, therefore, this particular model of the modified Cobb-Douglas function was rejected.

APPENDIX B

Estimation of Autoregressive Coefficient when One Has Two Series with Common Time Means*

Model:

$$Y_{ijt} = X_{jt} + u_{ijt} \qquad \begin{matrix}(i = 1, 2)\\ (j = 1, 2, \ldots, m)\\ (t = 1, 2, \ldots, n)\end{matrix}$$

where:

(a) i = index of lots treated alike

(b) j = index of different treatments

(c) t = index of time

(d) X_{jt} = the mean gain of lots treated alike at time t, hence, $X_{j0} = 0$

(e) Y_{ijt} = observed gain of the ith lot for treatment j at time t

(f) $u_{ijt} = \beta u_{ijt-1}$

(g) β = the autoregressive coefficient

(h) $e_{ijt} \sim$ N.I.D.$(0, \sigma^2)$ $\qquad (t > 0)$

(i) $e_{ij0} = 0$ (i.e., assume the first observation is zero and measured without error).

The likelihood can be written as:

$$L = \left[\frac{1}{2\pi\sigma^2}\right]^{n/2} \cdot e^{-\frac{\sum_i \sum_j \sum_t \left[Y_{ijt} - X_{jt} - \beta(Y_{ijt-1} - X_{jt-1})\right]^2}{2\sigma^2}}$$

---

*The author is indebted to Wayne A. Fuller for this proof that the estimate of the autoregressive coefficient, β, is a maximum likelihood estimate.

then

(103a)

$$\frac{\partial \log L}{\partial X_{11}} = \frac{-2}{2\sigma^2}\left[\sum_i \left[Y_{i11} - X_{11} - \beta(Y_{i10} - X_{10})\right](-1)\right] = 0$$

(103b) $\frac{\partial \log L}{\partial X_{12}} = \ldots$

$\vdots$

and

(104) $$\frac{\partial \log L}{\partial \beta} = \frac{-2}{2\sigma^2}\left[\sum_i \sum_j \sum_t \left[\left[Y_{ijt} - X_{jt} - \beta(Y_{ijt-1} - X_{jt-1})\right](Y_{ijt-1} - X_{jt-1})(-1)\right]\right] = 0$$

(105) $$\frac{\partial \log L}{\partial \sigma^2} = \frac{-n}{2\sigma^2} + \frac{2\sum_i \sum_j \sum_t e^2_{ijt}}{4\sigma^4} = 0$$

From equations 103a, 103b, ... the following relationships may be obtained:

(106a) $\bar{y}_{\cdot 11} - X_{11} - \beta(\bar{y}_{\cdot 10} - X_{10}) = 0$

(106b) $\bar{y}_{\cdot 12} - X_{12} - \beta(\bar{y}_{\cdot 11} - X_{11}) = 0$

$\vdots$

since X_{j0} and $\bar{y}_{\cdot j0} = 0$

then

(107a) $$\hat{X}_{11} = \bar{y}_{\cdot 11} = \frac{\sum_{i} Y_{i11}}{2}$$

(107b) $$\hat{X}_{12} = \bar{y}_{\cdot 12}$$

$$\vdots$$

thus

(108) $$\hat{\beta} = \frac{\sum_{i} \sum_{j} \sum_{t} (Y_{ijt} - \bar{y}_{\cdot jt})(Y_{ijt-1} - \bar{y}_{\cdot jt-1})}{\sum_{i} \sum_{j} \sum_{t} (Y_{ijt-1} - \bar{y}_{\cdot jt-1})^2}$$

and

(109) $$\hat{\sigma}^2 = \frac{\sum_{i} \sum_{j} \sum_{t} \hat{e}^2_{ijt}}{n}$$

where

(110) $$\hat{e}_{ijt} = Y_{ijt} - \bar{y}_{\cdot jt} - \beta(Y_{ijt-1} - \bar{y}_{\cdot jt-1}).$$

Thus the estimate of β is a maximum likelihood estimate.

APPENDIX C

The Aggregate Production Function*

The aggregate production function as presented in this section is based on the same statistical assumptions as the overall stilbestrol and the overall non-stilbestrol functions presented in an earlier section. Furthermore, all the variables used in the aggregate function are defined and measured in the same manner as with the overall functions (see page 56 ff.).

---

*The aggregate production function has been tested against the overall stilbestrol production function and the overall non-stilbestrol production function to determine if there is a difference between the overall stilbestrol and the overall non-stilbestrol production functions. The following F test was used:

$$F = \frac{\dfrac{SSE_{ag\ ftn} - SSE_{os\ ftn} - SSE_{on\ ftn}}{df_{ag\ ftn} - df_{os\ ftn} - df_{on\ ftn}}}{\dfrac{SSE_{os\ ftn} + SSE_{on\ ftn}}{df_{os\ ftn} + df_{on\ ftn}}}$$

where: SSE = sum of squares for error
df = degrees of freedom
ag ftn = aggregrate production function
os ftn = overall stilbestrol production function
on ftn = overall non-stilbestrol production function.

The computed value of F with 6 and 276 degrees of freedom is:

$$F^{6}_{276} = 2.3448.$$

The table values for F with 6 and 276 degrees of freedom are approximately:

(footnote continued on next page)

The estimated aggregate production function is:

$$(111) \quad G = .13628727C + .02193828F - .00000819C^2 - .00000063F^2 - .00000253CF - 1.75011550H.$$

The coefficient of determination, standard errors and the "t" values for the aggregate production function are presented in Table 58.

Table 58. Coefficient of determination, standard errors and "t" values for the aggregate production function (equation 111)

R^2	Independent variable	Standard error of regression coefficient	"t" value	Level of significance
.9725	C	.01144547	11.908	$p < .001$
	F	.00157833	13.900	$p < .001$
	C^2	.00000404	2.027	$.025 < p < .05$
	F^2	.00000011	5.727	$p < .001$
	CF	.00000128	1.977	$.05 < p < .10$
	H	.22004461	7.954	$p < .001$

(footnote continued from previous page)

$$F^{6}_{276} \simeq 2.13 \text{ (the 5\% level)}$$

$$F^{6}_{276} \simeq 2.87 \text{ (the 1\% level)}.$$

Therefore, at the 5 per cent probability level there is reason to believe that there is a difference between the overall stilbestrol and the overall non-stilbestrol production functions. However, at the 1 per cent probability level this disparity between the two functions is no longer significant.

The beef gain isoquant equation, as derived from the aggregate production function, is as follows:

$$(112)\quad F = 17{,}411.33333 - 2.00793651C \pm (-793{,}650.793)\Big[(.02193828 - .00000253C)^2 - .00000252(.13628727C - .00000819C^2 - 1.75011550H - G)\Big]^{1/2}.$$

The equation for predicting the marginal rates of substitution of corn for soilage is:

$$(113)\quad \frac{\partial F}{\partial C} = \frac{.13628727 - .000016380C - .00000253F}{.02193828 - .00000126F - .00000253C}.$$

The beef gain isoquant schedules and the marginal rates of substitution associated with them have been derived for 100, 200, 300, 350 and 400 pounds of beef gain and are presented in Table 59. The above isoquant schedules are depicted in Figure 27.

The prediction equation for estimating the quantities of corn and soilage that are required to produce various levels of gain for different soilage-corn rations is derived from the aggregate production function and the ration equation $\frac{F}{C} = \alpha$ (see page 75 ff. for further information on the derivation procedure).

The derived equation for predicting the quantities of corn that are required to produce various levels of gain for various soilage-corn rations is:

Table 59. Isoquant schedules, derived from the aggregate quadratic function, showi
marginal rates of substitution of corn for soilage at five gain levels,
steers (temperature is held constant at the overall mean)

Lbs. corn	100 lbs. gain: Lbs. soilage	100 lbs. gain: Ration[c]	100 lbs. gain: $\frac{\partial F}{\partial C}$[b]	200 lbs. gain: Lbs. soilage	200 lbs. gain: Ration	200 lbs. gain: $\frac{\partial F}{\partial C}$	300 lbs. gain: Lbs. soilage	300 lbs. gain: Ration	300 lbs. gain: $\frac{\partial}{\partial}$
0	5,394	--[d]	8.10						
100	4,602	46.02	7.74	16,715	169.15	148.01			
200	3,844	19.22	7.43	13,187	65.94	20.69			
300	3,114	10.38	7.16	11,442	38.14	15.15			
400	2,411	6.03	6.91	10,064	25.16	12.65			
500	1,731	3.46	6.69	8,880	17.76	11.14			
600	1,072	1.79	6.49	7,821	13.04	10.10			
700				6,[illegible]2	[illegible]	[illegible].32			
800				5,952	7.44	8.71			
900				5,107	5.67	8.21			
1,000				4,307	4.31	7.[illegible]0			
1,100				3,565	3.22	7.44			
1,200				2,817	2.35	[illegible].13			
1,300				2,118	1.63	6.86	13,339	10.2[illegible]	[illegible]
1,400							10,811	7.72	1[illegible]
1,500							9,262	6.17	13.[illegible]
1,600							[illegible],013	5.01	11.[illegible]
1,700							[illegible],831	4.0[illegible]	10.[illegible]
1,800							[illegible]	3.31	[illegible]
1,900							5,0[illegible]6	2.67	[illegible]
2,000							4,235	2.12	[illegible]
2,100							3,455	1.65	7.[illegible]
2,200									
2,300									
2,400									
2,500									
2,600									
2,700									
2,800									
2,900									
3,000									

[a]For each of the feed combinations there would also be fed a certain amount of t
be fed at the rate of .2 of a pound per day.

[b]The marginal rate of substitution of corn for soilage.

[c]Ration is the ratio of soilage to corn.

[d]The all soilage ration.

function, showing po sible feed combinations[a] and
ve gain levels, for 550 pound good-to-choice feeder
)

300 lbs. gain			350 lbs. gain			400 lbs. gain		
. age	Ration	$\frac{\partial F}{\partial C}$	Lbs. scilage	Ration	$\frac{\partial F}{\partial C}$	Lbs. scilage	Ration	$\frac{\partial F}{\partial C}$
9	10.2[illegible]	[illegible].11						
1	7.72	1[illegible].01						
2	6.17	13.[illegible]						
3	5.01	11.52						
1	4.0[illegible]	10.21						
[illegible]	3.31	[illegible].29						
6	2.67	[illegible]						
5	2.12	[illegible].04	11,265	5.63	27.94			
5	1.65	7.5[illegible]	[illegible],257	4.41	15.82			
			7,867	3.58	12.44			
			6,719	2.92	10.66			
			5,713	2.33	9.52			
			4,804	1.92	8.70			
						8,985	3.21	19.16
						7,418	2.56	13.32
						6,217	2.07	10.96

tain amount of the supplement shown in Table 3. This supplement would

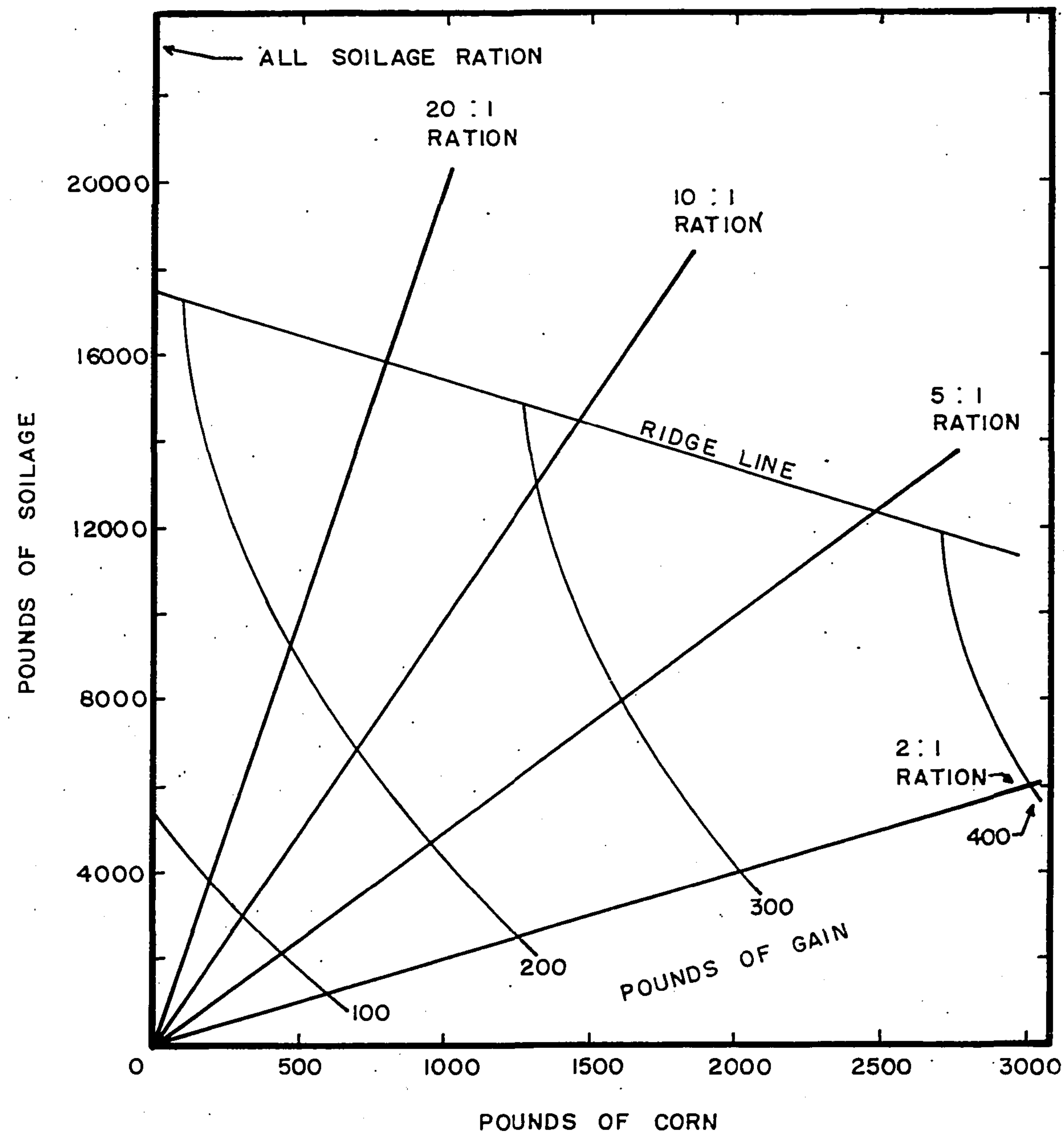

Figure 27. Gain isoquants and selected ration lines for the aggregate production function (temperature held constant at the overall mean)

$$(114)\quad C = -(.13628727 + .02193828\alpha)(-.00001638 - .00000506\alpha - .00000126\alpha^2)^{-1} \pm (-.00001638 - .00000506\alpha - .00000126\alpha^2)^{-1} \left[(.13628727 + .02193828\alpha)^2 - (-.00003276 - .00001012\alpha - .00000252\alpha^2)(-1.75011550H - G)\right]^{1/2}.$$

Once the corn values for any given ration have been determined, the corresponding soilage values are readily determined with the ration equation $F = \alpha C$.

The predicted quantities of corn and soilage, for selected rations at various levels of gain (*i.e.*, 100, 200, 300, 350, and 400 pounds) and the associated marginal rates of substitution of corn for soilage are presented in Table 60. The ration lines corresponding to the data in Table 60 have been plotted in Figure 27.

Ration lines

Total and marginal gain equations, for eight selected rations, are derived from the aggregate production function and are shown in Table 61 (see page 79 ff. for more information on the derivation of the ration equations). The predicted total gain values for various levels of feed input, for the eight selected rations, are shown in Table 62 and plotted in Figure 28. The estimated marginal gain values corresponding to the total gain values are presented in Table 63.

Table 60. Corn and soilage quantities[a] and the marginal rate of substitution alon
for selected rations (temperature is held constant at the overall mean)

Ration (ratio of soilage to corn)	100 lbs. gain			200 lbs. gain			300 lbs. g	
	Lbs.[b] soilage	Lbs.[c] corn	$\frac{\partial F}{\partial C}$[d]	Lbs. soilage	Lbs. corn	$\frac{\partial F}{\partial C}$	Lbs. soilage	Lbs. corn
All soilage	5,394	--	8.10					
20:1	3,886	194	7.45	9,286	464	11.61		
15:1	3,565	238	7.32	8,302	553	10.54		
10:1	3,067	307	7.14	6,924	692	9.37	13,067	1,307
8:1	2,779	347	7.04	6,186	773	8.86	11,083	1,385
5:1	2,173	435	6.83	4,731	946	8.01	8,004	1,601
3:1	1,572	524	6.64	3,371	1,123	7.36	5,538	1,846
2:1	1,170	585	6.52	2,492	1,246	7.00	4,047	2,024

[a]For each of the feed combinations there would also be fed a certain amount of
be fed at the rate of .2 of a pound per day.

[b]The all soilage value was derived from equation 112, all other values were de

[c]Derived from equation 114.

[d]The marginal rate of substitution of corn for soilage.

on along the 100, 200, 300, 350 and 400 pound beef gain isoquants
ll mean)

) lbs. gain		350 lbs. gain			400 lbs. gain		
Lbs. corn	$\frac{\partial F}{\partial C}$	Lbs. scilage	Lbs. corn	$\frac{\partial F}{\partial C}$	Lbs. scilage	Lbs. corn	$\frac{\partial F}{\partial C}$
1,307	37.74						
1,385	19.14						
1,601	11.51	10,225	2,045	19.82			
1,846	8.94	6,861	2,287	10.85	8,484	2,828	16.73
2,024	7.93	4,964	2,481	8.83	6,034	3,017	10.68

mount of the supplement shown in Table 3. This supplement would

were derived from the ration equation $F = \propto C$.

Table 61. Total and marginal gain equations, derived from aggregate production function, for selected rations for 850 pound good-to-choice feeder steers

Ration[a]	Prediction equations for: Total gain	Marginal gain
Ration A All soilage	$G_A = .02193828\,\gamma_A{}^{b} - .00000063\,\gamma_A^2 - 1.75011550H$	$\frac{\partial G_A}{\partial \gamma_A} = .02193828 - .00000126\,\gamma_A$
Ration B 20:1	$G_B = .02738347\,\gamma_B - .00000070\,\gamma_B^2 - 1.75011550H$	$\frac{\partial G_B}{\partial \gamma_B} = .02738347 - .00000140\,\gamma_B$
Ration C 15:1	$G_C = .02908509\,\gamma_C - .00000073\,\gamma_C^2 - 1.75011550H$	$\frac{\partial G_C}{\partial \gamma_C} = .02908509 - .00000146\,\gamma_C$
Ration D 10:1	$G_D = .03233364\,\gamma_D - .00000080\,\gamma_D^2 - 1.75011550H$	$\frac{\partial G_D}{\partial \gamma_D} = .03233364 - .00000160\,\gamma_D$
Ration E 8:1	$G_E = .03464372\,\gamma_E - .00000085\,\gamma_E^2 - 1.75011550H$	$\frac{\partial G_E}{\partial \gamma_E} = .03464372 - .00000170\,\gamma_E$
Ration F 5:1	$G_F = .04099645\,\gamma_F - .00000102\,\gamma_F^2 - 1.75011550H$	$\frac{\partial G_F}{\partial \gamma_F} = .04099645 - .00000204\,\gamma_F$
Ration G 3:1	$G_G = .05052553\,\gamma_G - .00000134\,\gamma_G^2 - 1.75011550H$	$\frac{\partial G_G}{\partial \gamma_G} = .05052553 - .00000268\,\gamma_G$
Ration H 2:1	$G_H = .06005461\,\gamma_H - .00000175\,\gamma_H^2 - 1.75011550H$	$\frac{\partial G_H}{\partial \gamma_H} = .06005461 - .00000350\,\gamma_H$

[a]Ration is the ratio of soilage to corn.

[b]γ denotes total pounds of feed of the particular ration.

Table 62. Estimated total gain from various total feed quantities[a] of selected soilage-corn rations fed to 850 pound good-to-choice feeder steers[b]

Pounds of feed fed	Total gain[c] in pounds for selected rations:[d] All soilage	20:1	15:1	10:1	8:1	5:1	3:1	2:1
500	10.8	13.5	14.4	16.0	17.1	20.2	24.9	29.6
1,000	21.3	26.7	28.4	31.5	33.8	40.0	49.2	58.3
1,500	31.5	39.5	42.0	46.7	50.1	59.2	72.8	86.1
2,000	41.4	51.9	55.2	61.5	65.9	77.9	95.7	113.1
2,500	50.9	64.1	68.1	75.9	81.3	96.1	117.9	139.2
3,000	60.1	75.8	80.6	89.8	96.3	113.8	139.5	164.4
3,500	69.1	87.2	92.8	103.4	110.9	131.0	160.4	188.7
4,000	77.7	98.3	104.6	116.6	125.0	147.7	180.7	212.2
4,500	86.0	109.0	116.0	129.4	138.7	163.9	200.2	234.8
5,000	93.9	119.3	127.1	141.7	152.0	179.6	219.1	256.5
5,500	101.6	129.3	137.8	153.7	164.9	194.7	237.3	277.3
6,000	108.9	138.9	148.1	165.3	177.3	209.4	254.9	297.2
6,500	116.0	148.2	158.0	176.5	189.3	223.5	271.8	316.3
7,000	122.7	157.2	167.6	187.3	200.9	237.2	288.0	334.5
7,500	129.1	165.7	176.9	197.6	212.1	250.3	303.5	351.8
8,000	135.2	174.0	185.7	207.6	222.8	262.9	318.4	368.3
8,500	141.0	181.8	194.2	217.2	233.1	275.0	332.6	383.9
9,000	146.4	189.4	202.3	226.4	243.0	286.6	346.1	398.6
9,500	151.6	196.5	210.1	235.2	252.5	297.7	359.0	412.4
10,000	156.4	203.4	217.5	243.6	261.6	308.3	371.2	425.3
10,500	160.9	209.8	224.5	251.6	270.2	318.4	382.7	437.4
11,000	165.1	215.9	231.1	259.2	278.4	328.0	393.6	448.6
11,500	169.0	221.7	237.4	266.4	286.2	337.0	403.7	458.9
12,000	172.5	227.1	243.3	273.2	293.5	345.6	413.3	468.3
12,500	175.8	232.2	248.9	279.6	300.4	353.6	422.1	476.9
13,000	178.7	236.9	254.1	285.6	306.9	361.2	430.3	484.6
13,500	181.3	241.2	258.9	291.2	313.0	368.2	437.8	
14,000	183.7	245.2	263.3	296.4	318.7	374.7		
14,500	185.6	248.9	267.4	301.2	323.9			
15,000	187.3	252.2	271.1	305.6				
15,500	188.7	255.1	274.5					

6,500	116.0	148.2	158.0	176.5	189.3	223.5	271.8	316.3
7,000	122.7	157.2	167.6	187.3	200.9	237.2	288.0	334.5
7,500	129.1	165.7	176.9	197.6	212.1	250.3	303.5	351.8
8,000	135.2	174.0	185.7	207.6	222.8	262.9	318.4	368.3
8,500	141.0	181.8	194.2	217.2	233.1	275.0	332.6	383.9
9,000	146.4	189.4	202.3	226.4	243.0	286.6	346.1	398.6
9,500	151.6	196.5	210.1	235.2	252.5	297.7	359.0	412.4
10,000	156.4	203.4	217.5	243.6	261.6	308.3	371.2	425.3
10,500	160.9	209.8	224.5	251.6	270.2	318.4	382.7	437.4
11,000	165.1	215.9	231.1	259.2	278.4	328.0	393.6	448.6
11,500	169.0	221.7	237.4	266.4	286.2	337.0	403.7	458.9
12,000	172.5	227.1	243.3	273.2	293.5	345.6	413.3	468.3
12,500	175.8	232.2	248.9	279.6	300.4	353.6	422.1	476.9
13,000	178.7	236.9	254.1	285.6	306.9	361.2	430.3	484.6
13,500	181.3	241.2	258.9	291.2	313.0	368.2	437.8	
14,000	183.7	245.2	263.3	296.4	318.7	374.7		
14,500	185.6	248.9	267.4	301.2	323.9			
15,000	187.3	252.2	271.1	305.6				
15,500	188.7	255.1	274.5					
16,000	189.7	257.7						
16,500	190.5							
17,000	190.9							

[a]In addition to the feed fed there would also be fed a certain amount of the supplement shown in Table 3. This supplement would be fed at the rate of .2 of a pound per day.

[b]Temperature is held constant at the overall mean.

[c]All values are derived from the equations in Table 61.

[d]The ration is the ratio of soilage to corn.

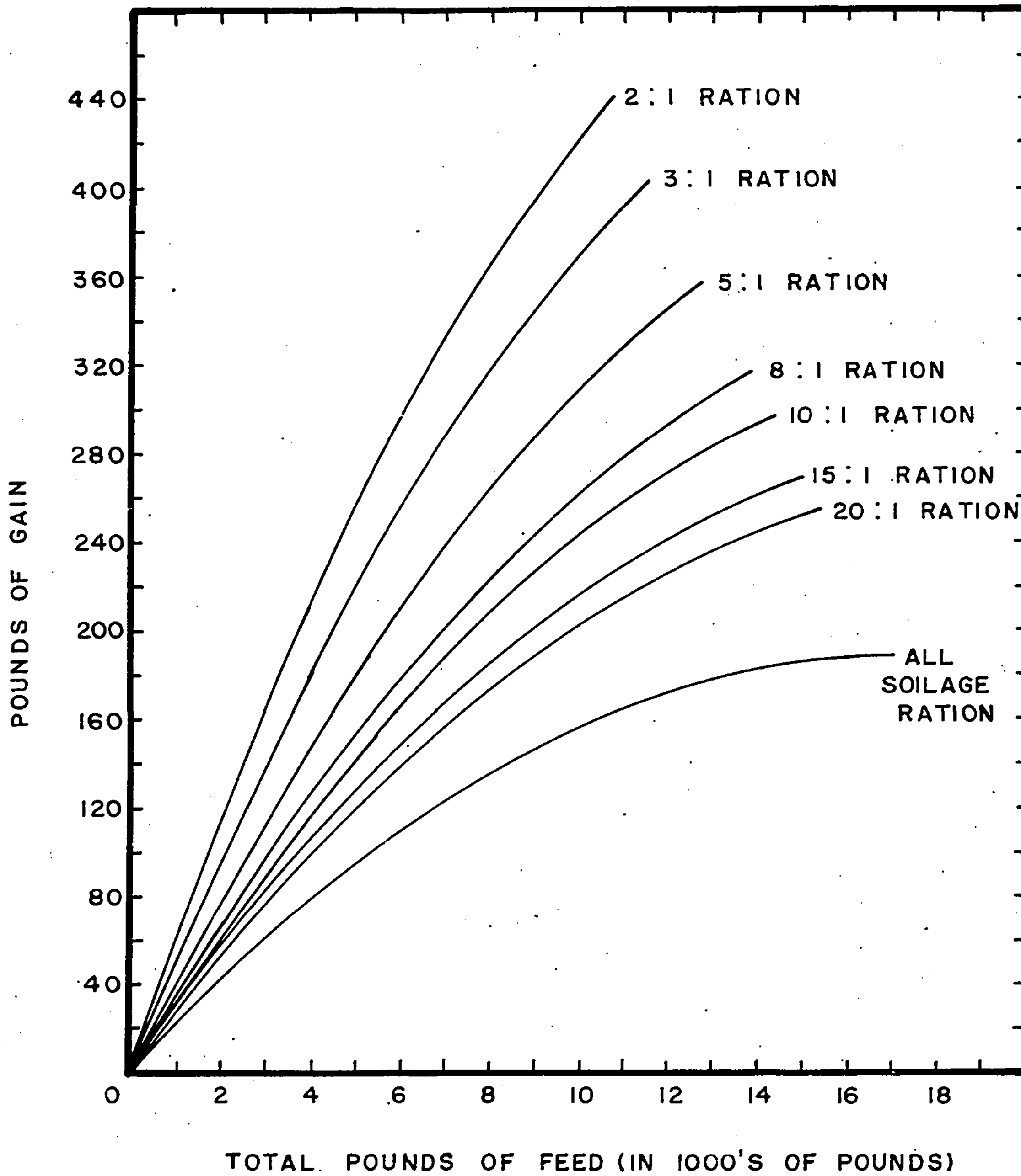

Figure 28. Feed-gain transformation curves for selected rations derived from the aggregate production function (temperature held constant at the overall mean)

Table 63. Estimated marginal gain from various total feed quantities of selected soilage-corn rations fed to 850 pound good-to-choice feeder steers

Pounds of feed fed	Marginal gain[a] in pounds for selected rations:[b]							
	All soilage	20:1	15:1	10:1	8:1	5:1	3:1	2:1
500	.0213	.0267	.0284	.0315	.0338	.0400	.0492	.0583
1,000	.0207	.0260	.0276	.0307	.0329	.0390	.0478	.0566
1,500	.0200	.0253	.0269	.0299	.0321	.0379	.0465	.0548
2,000	.0194	.0246	.0261	.0291	.0312	.0369	.0452	.0530
2,500	.0188	.0239	.0254	.0283	.0304	.0359	.0438	.0513
3,000	.0182	.0232	.0247	.0275	.0296	.0349	.0425	.0495
3,500	.0175	.0225	.0239	.0268	.0287	.0339	.0411	.0478
4,000	.0169	.0217	.0232	.0260	.0279	.0329	.0398	.0460
4,500	.0163	.0210	.0225	.0252	.0270	.0318	.0385	.0443
5,000	.0156	.0203	.0217	.0244	.0262	.0308	.0371	.0425
5,500	.0150	.0196	.0210	.0236	.0253	.0298	.0358	.0408
6,000	.0144	.0189	.0203	.0228	.0245	.0288	.0344	.0390
6,500	.0137	.0182	.0195	.0220	.0236	.0278	.0331	.0373
7,000	.0131	.0175	.0188	.0212	.0228	.0268	.0318	.0355
7,500	.0125	.0168	.0181	.0204	.0219	.0258	.0304	.0338
8,000	.0119	.0161	.0173	.0196	.0211	.0247	.0291	.0320
8,500	.0112	.0154	.0166	.0188	.0202	.0237	.0277	.0303
9,000	.0106	.0147	.0159	.0180	.0194	.0227	.0264	.0285
9,500	.0100	.0140	.0151	.0172	.0185	.0217	.0251	.0268
10,000	.0093	.0133	.0144	.0164	.0177	.0207	.0237	.0250
10,500	.0087	.0126	.0137	.0156	.0168	.0197	.0224	.0233
11,000	.0081	.0119	.0129	.0148	.0160	.0186	.0210	.0215
11,500	.0074	.0112	.0122	.0140	.0151	.0176	.0197	.0198
12,000	.0068	.0105	.0115	.0132	.0143	.0166	.0184	.0180
12,500	.0062	.0098	.0107	.0124	.0134	.0156	.0170	.0162
13,000	.0056	.0091	.0100	.0116	.0126	.0146	.0157	.0145
13,500	.0049	.0084	.0093	.0108	.0117	.0136	.0143	
14,000	.0043	.0077	.0085	.0100	.0109	.0125		
14,500	.0037	.0069	.0077	.0092	.0100			
15,000	.0030	.0062	.0071	.0084				
15,500	.0024	.0055	.0063					
16,000	.0018	.0048						
16,500	.0011							
17,000	.0005							

[a]All values are derived from the equations in Table 61.

[b]The ration is the ratio of soilage to corn.